> # Maple Program Ring3  Appendix 2
> # One-step calculation of a 3-state Ring model
> restart;
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> M3ring:=matrix(3,3,[[m12, 0, m31],[m12, -m23, 0],[0, m23, -m31]]);

\[ M_{3ring} := \begin{bmatrix}
-m12 & 0 & m31 \\
m12 & -m23 & 0 \\
0 & m23 & -m31
\end{bmatrix} \]

> z:=m31/m23;

\[ z := \frac{m31}{m23} \]

> # Setting the known initial proportions, w, and interval length n
> f0:=.35; g0:=.2; n:=1; w:=.75;

\[ f0 := .35 \]
\[ g0 := .2 \]
\[ n := 1 \]
\[ w := .75 \]

> # Setting the value of rate ratio z
> z:=.033;

\[ z := .033 \]

> # Simultaneously solving the 2 IL and 2 flow equations
> # The dominant right eigenvector is \([1, m12/m23, m12/m31]\)
> # The associated tau vector is
> \([m23*m31/(m23*m31+m12*m31+m12*m23), m12*m31/(m23*m31+m12*m31+m12*m23), m12*m23/(m23*m31+m12*m31+m12*m23)]\)

> solve( { f1 = z*(1-w)/(1+z*z*m23/m12) + w*f0 , g1 = (1-w)/(1+z*z*m23/m12) + w*g0 , f1 = f0 - (n/2)*(f0+f1)*m23 + (n/2)*(2-f0-g0-f1-g1)*m12 , g1 = g0 - (n/2)*(g0+g1)*m23*z + (n/2)*(f0+f1)*m23 , };

\{m23 = .2861326497, f1 = .2669871984, g1 = .2859757079, m12 = .01172145356 \}
\{m12 = .2502770376, m23 = .5832277130, f1 = .2699330965, g1 = .3752453471 \}

> # The set values yield 2 demographically valid solutions.
Choosing the first solution
> m12:= .1172145356e-1; f1:= .2669871984; g1:= .2859757079; m23:= .2861326497;

m12 := .01172145356

f1 := .2669871984

g1 := .2859757079

m23 := .2861326497

> # Solving for the third rate
> m31 := z*m23;

    m31 := .009442377440

> # Using the linear assumption to calculate the projection matrix
    PI
> I3:=matrix(3,3,[[1,0,0],[0,1,0],[0,0,1]]);

        I3 :=
            [ 1 0 0 ]
            [ 0 1 0 ]
            [ 0 0 1 ]

> IplusM:=evalm(I3+(n/2)*M3ring):
> IminusM:=evalm(I3-(n/2)*M3ring):
> IminusMinv:=inverse(IminusM):
> PI:=evalm(IminusMinv*IplusM);

Pi :=
        .9883536560   .001169406281   .009343281457
    .01019468186   .7496857449   .00004790484913
        .001451662096   .2491448494   .9906088141

> # Verifying the projection relationship, from initial population
x0 to ending population x1
> x0:=matrix(3,1,[[1-f0-g0],[f0],[g0]]);

x0 := [.45 .35 .2]

> x1:=evalm(PI*x0);

    x1 :=
        [.4470370937]
        .2669871985
        .2859757080

> # Projection checks to calculated values of f1 and g1
# Maple Program Path3

## Two-step calculation of a 3-state path model

```maple
restart;

with(linalg):

Warning, new definition for norm
Warning, new definition for trace

M3path := matrix(3, 3, [[-m12, m21, 0], [m12, -m21 - m32, m32], [0, m23, -m32]]);

\[
M3path := \begin{bmatrix}
-m12 & m21 & 0 \\
m12 & -m21 - m32 & m32 \\
0 & m23 & -m32 \\
\end{bmatrix}
\]

z1 := m12/m21; z2 := m23/m32;

z1 := \frac{m12}{m21}

z2 := \frac{m23}{m32}

# Setting the known initial values, w, and interval length n
f0 := .35; g0 := .25; n := 1; w := .75;

f0 := .35

\[g0 := .25\]

n := 1

\[w := .75\]

# Setting the values of rate ratios z1 and z2
z1 := 2.5; z2 := 1.3;

\[z1 := 2.5\]

\[z2 := 1.3\]

# Solving the 2 IL equations and then the 2 flow equations

# The dominant right eigenvector is [1, m12/m21, (m12/m21)*(m23/m32)]' or [1, z1, z1*z2]'

# The associated tau vector is [1/(1+z1+z1*z2), z1/(1+z1+z1*z2), z1*z2/(1+z1+z1*z2)]'

solve( { f1 = z1*(1-w)/(1+z1+z1*z2) + w*f0 ,

\[g1 = z1*z2*(1-w)/(1+z1+z1*z2) + w*g0 \},

\{f1, g1\});

\{f1 = 0.3550925926, g1 = 0.3078703704\}

# Using the result for the end of interval populations

# to solve for 2 transfer rates

f1 := .3550925926; g1 := .3078703704;

\[f1 := 0.3550925926\]

\[g1 := 0.3078703704\]
```

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> solve( { f1 = f0 - (n/2)*(f0+f1)*(m21+m32*z2) +
(n/2)*(2-f0-g0-f1-g1)*m21*z1 + (n/2)*(g0+g1)*m32 ,
> g1 = g0 - (n/2)*(g0+g1)*m32 + (n/2)*(f0+f1)*m32*z2},
> {m21, m32} );
> {m21 = .1107041108, m32 = .3226222740}

> # The given values yield a valid solution. Hence
> m21 := .1107041108; m32 := .3226222740;
> m21 := .1107041108
> m32 := .3226222740

> # Solving for the other 2 rates
> m12 := z1*m21; m23 := z2*m32;
> m12 := .2767602770
> m23 := .4194089562

> # Using the linear assumption to calculate the projection matrix
> PI
> I3 := matrix(3,3,[[1,0,0],[0,1,0],[0,0,1]]);
> I3 :=
> \[
> \begin{bmatrix}
> 1 & 0 & 0 \\
> 0 & 1 & 0 \\
> 0 & 0 & 1 \\
> \end{bmatrix}
> \]
> IplusM := evalm(I3+(n/2)*M3path):
> IminusM := evalm(I3-(n/2)*M3path):
> IminusMinv := inverse(IminusM):
> PI := evalm(IminusMinv*IplusM);
> PI :=
> \[
> \begin{bmatrix}
> .7664993963 & .07911415403 & .01098929799 \\
> .1977853852 & .6270756525 & .2260078417 \\
> .03571521847 & .2938101941 & .7630028607 \\
> \end{bmatrix}
> \]

> # Verifying the projection relationship, from initial population
> x0 to ending population x1
> x0 := matrix(3,1,[[1-f0-g0],[f0],[g0]]);
> x0 :=
> \[
> \begin{bmatrix}
> .40 \\
> .35 \\
> .25 \\
> \end{bmatrix}
> \]
> x1 := evalm(PI*x0);
> x1 :=
> \[
> \begin{bmatrix}
> .3370370369 \\
> .3550925929 \\
> .3078703705 \\
> \end{bmatrix}
> \]

> # Projection checks to calculated values of f1 and g1
>
# Maple Program Tri3voting

Calculating an N=3 Triangular Model

```maple
restart;
with(linalg):
Warning, new definition for norm
Warning, new definition for trace

# The program carries out calculations for Table 3, finding
# rates of transfer between voting statuses for 2002-2006
# Entering the initial (2002) proportions in states L, P, and N
L0 := .395; P0 := .147; N0 := .458;

L0 := .395
P0 := .147
N0 := .458

# The data values for the ending (2006) proportions
L1 := .404; P1 := .197; N1 := .399;

L1 := .404
P1 := .197
N1 := .399

# Setting the IL parameter and the interval length
w := .8; n := 4;

w := .8
n := 4

# Simultaneously solving the 2 IL and 2 flow equations
# The dominant right eigenvector is \( \lambda \) [ (mPL + mPN)/mLP, 1, mPN/mNL ]

solve( { P1 = (1-w)*(1/(1+ mPN/mNL+ (mPL+mPN)/mLP)) + w*P0 ,
> N1 = (1-w)*((mPN/mNL)/(1+ mPN/mNL+ (mPL+mPN)/mLP)) +
> w*N0 ,
> P1 = P0 - (n/2)*(P0+P1)*(mPL+mPN) +
(n/2)*(2-P0-P1-N0-N1)*mLP ,
> N1 = N0 - (n/2)*(N0+N1)*mNL + (n/2)*(P0+P1)*mPN } ,
> {mLP = .04121489177, mLP = .05984575773, mPN = .01692198327, mPL = .04940580868}
```