Additional file 2: Cs1 requirements to take large negative values

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The Coefficient of Sociality (Cs) compares the mean distance between simultaneous pairs of fixes ($D_O$) against the mean distance between all permutations of all fixes ($D_E$).

$$
Cs = \frac{D_E - D_O}{D_E + D_O} = 1 - 2 \frac{D_O}{D_E + D_O},
$$

(1)

where

$$
D_O = \left( \sum_{i=1}^{T} d_{A,B}^i \right) / T,
$$

and

$$
D_E = \left( \sum_{i=1}^{T} \sum_{t_1=1}^{T} d_{A,B}^{i_1,i_2} / T^2. 
$$

Let $d_{ij}$ be the distance between the locations of A at time $i$ and B at time $j$. Then, $D_O$ and $D_E$ can be expressed as in equations 2 and 4.

$$
D_O = \sum_{i_1=1}^{T} d_{ii} / T
$$

(2)

$$
D_O = \sum_{i,j \in [1,T]} \frac{d_{ij}}{(T^2 - T)}
$$

(3)
\[ D_E = \frac{D_O}{T} + \frac{(T-1)}{T}D_O \]  (4)

where \( D_O \) is defined in equation 3 and corresponds to the average distance between the exclusively permuted points without taking into account the simultaneous fixes. Using those equations, we can replace \( D_O \) and \( D_E \) in equation 1 when \( Cs1 = -\alpha \) (\( \alpha > 0 \)) and obtain:

\[
\frac{D_O}{D_O} = \frac{T(1-\alpha)}{(T-1)(1+\alpha)} - \frac{1}{T-1} \]  (5)

It means that, for instance, for \( Cs1 = -0.5 \) and when \( T \) is large, \( D_O \) would have to be approximately a third of \( D_O \), thus a third of the average distance computed only at simultaneous fixes. Fig. 1 shows the values of \( D_O/D_O \) ratios needed to attain the whole range of \( Cs \) negative values. Most of those scenarios are very unlikely.
Figure 1: Computed ratios $D_O/D_O$ needed for obtaining the Cs1 negative values (x-axis, from −0.99 to 0) for each series length $T$ (y-axis, from 2 to 250). The blank spaces correspond to infeasible situations.