Using the Gaussian kernel and computing the expected value over a window with $N$ samples, one could write:

$$J_n = \frac{1}{2\pi\sigma^2} \frac{1}{N} \sum_{i=n-N+1}^{n} \exp \left( -\frac{(d_i - y_i)(d_i - y_i)^*}{2\sigma^2} \right) = \frac{1}{2\pi\sigma^2} \frac{1}{N} \sum_{i=n-N+1}^{n} \exp \left( -\frac{e_i e_i^*}{2\sigma^2} \right)$$

Expanding the exponential argument, $e_i e_i^*$:

$$e_i e_i^* = (d_i - y_i)(d_i - y_i)^*$$
$$e_i e_i^* = (d_i - y_i)(d_i - y_i)^*$$
$$e_i e_i^* = (d_i^T d_i^T - d_i y_i^* - y_i d_i^* + y_i y_i^*)$$

Knowing that $y_i = w^T x_i$

$$y_i = w^T x_i$$
$$y_i^* = (w^T x_i)^*$$
$$y_i^* = (w^T)^* (x_i)^*$$

But, $w^T = (w^T)^T = (w^T)^*$, then:

$$y_i^* = (w^T)^* (x_i)^*$$
$$y_i^* = (w^T)^* (x_i)^*$$

Then, one could rewrite (41) to achieve:

$$e_i e_i^* = (d_i d_i^* - d_i y_i^* - y_i d_i^* + y_i y_i^*)$$
$$e_i e_i^* = (d_i d_i^* - d_i y_i^* - y_i d_i^* + y_i y_i^*)$$

In order to use the ascend gradient, one could do:

$$W_{n+1} = W_n + \mu \nabla J_n$$

Then, using Wirtinger calculus to obtain the gradient:

$$\nabla J_n = \frac{\partial J_n}{\partial w} = \frac{1}{2\pi\sigma^2} \frac{1}{N} \sum_{i=n-N+1}^{n} \exp \left( -\frac{(d_i - y_i)(d_i - y_i)^*}{2\sigma^2} \right) \frac{(-1) \partial (e_i e_i^*)}{\partial w}$$

Then, obtaining the derivation of the exponential argument:

$$\frac{\partial (e_i e_i^*)}{\partial w} = \frac{\partial (d_i d_i^* - d_i y_i^* - y_i d_i^* + y_i y_i^*)}{\partial w}$$

which could be rewrite as:

$$\frac{\partial (e_i e_i^*)}{\partial w} = \frac{\partial (d_i d_i^*)}{\partial w} - \frac{\partial (d_i w^T x_i^*)}{\partial w} - \frac{\partial (w^T x_i d_i^*)}{\partial w} + \frac{\partial (w^T x_i w^T x_i^*)}{\partial w}$$
\[
\frac{\partial (e_i e_i^*)}{\partial w^*} = \frac{\partial (d_i d_i^*)}{\partial w^*} - \frac{\partial (d_i w^T x_i^*)}{\partial w^*} + \frac{\partial ((w^*)^T x_i^* w^T x_i^*)}{\partial w^*}
\]

Resulting in

\[
\frac{\partial (e_i e_i^*)}{\partial w^*} = 0 - 0 - x_i d_i^* + x_i w^T x_i^* \\
= -x_i d_i^* + x_i w^T x_i^* \\
= -(d_i^* - w^T x_i^*) x_i \\
= -(d_i^* - (w^T x_i^*) x_i \\
= -(d_i - w^T x_i^*) x_i \\
= -(d_i - w^H x_i) x_i \\
= -(d_i - y_i) x_i = -e_i x_i
\]

always using the denominator layout. Then, Eq. 46 could be written as

\[
\triangledown J_n = \frac{\partial J_n}{\partial w^*} = \frac{1}{4\pi \sigma^2} \frac{1}{N} \sum_{i=n-N+1}^{N} \exp\left(-\frac{e_i e_i^*}{2\sigma^2}\right) e_i^* X_i
\]

Which implies in a update rule:

\[
W_{n+1} = W_n + \mu N 4\pi \sigma^2 \frac{1}{N} \sum_{i=n-N+1}^{N} \exp\left(-\frac{e_i e_i^*}{2\sigma^2}\right) e_i^* X_i
\]

where \(e_i = d_i - W_n^H X_i\). Using the stochastic gradient, one could obtain

\[
W_{n+1} = W_n + \mu N 4\pi \sigma^2 \exp\left(-\frac{e_i e_i^*}{2\sigma^2}\right) e_i^* X_i
\]

finishing the demonstration.