Additional Files

AF1.1 CRBs for the NLOS Rayleigh Fading Model when sampling is performed at the chip rate

Expression in (36) admits further simplifications when the sampling is performed at the chip rate. If it is the case, Gs in (19) becomes the identity matrix I, and the roll-off factor may be discarded, reducing the number of parameters required to compute the Fisher matrix, as it is shown in Supplementary Table 1. Hence (36) becomes (63), and (38) becomes (64).

\[
F_\Psi = \sum_{k=1}^{N_s} \sum_{k_1=1}^{K} G_{k,k_1} J_\Psi G_{k,k_1}^T + C_1 e^{(5)}_{N_p} e^{(5)}_{N_p}^T + \sum_{q_1=1}^{2N} \sum_{q_2=1}^{2N} C_2^{(q_1,q_2)} e^{(5+q_1)}_{N_p} e^{(5+q_2)}_{N_p}^T
\]  

(63)

\[
\Psi' = \left[ k_0, \lambda_n, \gamma_{k, k_1}, \sigma_w^2 \right]^T G_{k,k_1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_\phi^{(k)} & \Lambda_t^{(k_1)} \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & P_s \frac{\partial \lambda^{(k)}}{\partial \Lambda_t^{(k_1)}} & 0 \\
0 & 0 & 0 & P_s \frac{\partial \lambda^{(k)}}{\partial \rho^{(k_1)}} & 0
\end{bmatrix} 
\]  

(64)

Table 1 Supplementary Table 1. Elements required in (47) to assemble the FIM in (63) for a Rayleigh Fading Channel. It corresponds to the modelling of a NLOS dispersed signal sampled at the chip rate.

<table>
<thead>
<tr>
<th>p</th>
<th>$\Psi_p$</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k_0$</td>
<td>$R_{\phi}$</td>
<td>$T_K$</td>
<td>$P_s \frac{\partial \Lambda_t}{\partial k_0}$</td>
</tr>
<tr>
<td>2</td>
<td>$\lambda_n$</td>
<td>$R_\phi$</td>
<td>$T_K$</td>
<td>$P_s \frac{\partial \Lambda_t}{\partial k_0}$</td>
</tr>
<tr>
<td>3</td>
<td>$P_s$</td>
<td>$R_\phi$</td>
<td>$T_K$</td>
<td>$\Lambda_t$</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma_w^2$</td>
<td>$I_{N_0}$</td>
<td>$I_K$</td>
<td>$I_N$</td>
</tr>
<tr>
<td>5</td>
<td>$\alpha$</td>
<td>$R_\phi$</td>
<td>$\frac{\partial T}{\partial \alpha}$</td>
<td>$P_s \Lambda_t$</td>
</tr>
<tr>
<td>6- $N_p$</td>
<td>$\rho_{p-5}$</td>
<td>$\frac{\partial R_\phi}{\partial \rho_{p-5}}$</td>
<td>$T_K$</td>
<td>$P_s \Lambda_t$</td>
</tr>
</tbody>
</table>

AF1.2 Asymptotic Expressions for Delay Estimates and a PCD Source

A closed expression for the asymptotic eigenvalues from matrix T for PCD sources is provided in 61 as:

\[
\lambda^{(k)}_{K} \approx \frac{1 - \alpha^2}{1 + \alpha^2 - 2\alpha \cos \left( \frac{k\pi}{K+1} \right)}
\]  

(65)

which allows us for computing analytic derivatives with respect to $\alpha$. For the asymptotic case, C1 in equation (36) is negligible and therefore FIM simplifies. This equation in (65) allows the computation of analytic derivatives with respect to $\alpha$. For the asymptotic case, C1 in equation (41) is negligible and FIM reduces to (66):
The FIM for delay estimates in case of fully coherent dispersed sources is given by:

\[
\mathbf{F}_\Psi 
\approx \sum_{k=1}^{N_s} \sum_{k_1=1}^{K} \mathbf{G}_{k,k_1} \mathbf{J}_\Psi \mathbf{G}_{k,k_1}^T
+ \sum_{q_1=1}^{2Nc} \sum_{q_2=1}^{2Nc} 2^{(q_1+q_2)} \mathbf{e}_{Np}^{(5+q_1)} \mathbf{e}_{Np}^{(5+q_2)}
\]

(66)

### AF1.3 CRB’s for Delay Estimates in case of Fully Coherent Dispersed Sources

The general expressions are useful for PCD sources, but in case of FCD sources they cannot be used since \( \mathbf{T} \) under this assumption just has one eigenvector different to zero and equal to \( K \), and another more suitable factorization must be done. Hence, in order to achieve adequate expressions for this special case, definitions of the terms described in (37) and (44) are modified as is shown in (67) and (68):

\[
\mathbf{T} = \mathbf{1}_K \mathbf{1}_K^T
\]

(67)

\[
\mathbf{R}_z^{-1} = \sum_{k=1}^{N_s} \mathbf{u}_\phi^{(k)} \mathbf{u}_\phi^{(k)H} \otimes \left[ \frac{1}{\sigma_w^2} \mathbf{P} \otimes \mathbf{I}_M + \frac{1}{K} \mathbf{1}_K \mathbf{1}_K^T \otimes \mathbf{R}_k^{-1} \right]
\]

(68)

\( \mathbf{P} \) and \( \mathbf{R}_k \) are given by equations (69) and (70) and \( \mathbf{1}_K \) is a vector of length \( K \) with all its components equal to one.

\[
\mathbf{P} = \mathbf{I}_K - \frac{1}{K} \mathbf{1}_K \mathbf{1}_K^T
\]

(69)

\[
\mathbf{R}_k = K \mathbf{P}_k \lambda_\phi^{(k)} \mathbf{A}_t + \sigma_w^2 \mathbf{I}_M
\]

(70)

The number of parameters for this case reduces as it can be seen in (71), and matrices \( \mathbf{A}, \mathbf{B} \) and \( \mathbf{C} \) used for derivatives are expressed in Supplementary Table 2.

\[
\Psi = [k_0, \lambda_n, r, \sigma_w^2, \rho]^T
\]

(71)

### Table 2

Supplementary Table 2. Elements required in (47) to assemble the FIM in (72) for FCD sources. It corresponds to the modelling of a NLOS Rayleigh fading channel sampled at the chip rate when channel estimates are fully coherent.

<table>
<thead>
<tr>
<th>p</th>
<th>( \Psi_p )</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( k_0 )</td>
<td>( \mathbf{R}_a )</td>
<td>( \mathbf{1}_K \mathbf{1}_K^T/K )</td>
<td>( K \mathbf{r}(\partial \Lambda_r / \partial k_0) )</td>
</tr>
<tr>
<td>2</td>
<td>( \lambda_n )</td>
<td>( \mathbf{R}_a )</td>
<td>( \mathbf{1}_K \mathbf{1}_K^T/K )</td>
<td>( K \mathbf{r}(\partial \Lambda_r / \partial k_0) )</td>
</tr>
<tr>
<td>3</td>
<td>( r )</td>
<td>( \mathbf{R}_a )</td>
<td>( \mathbf{1}_K \mathbf{1}_K^T/K )</td>
<td>( \Lambda_r )</td>
</tr>
<tr>
<td>4</td>
<td>( \sigma_w^2 )</td>
<td>( \mathbf{I}_{N_p} )</td>
<td>( \mathbf{1}_K )</td>
<td>( \mathbf{I}_N )</td>
</tr>
<tr>
<td>5</td>
<td>( N_p )</td>
<td>( \rho_p )</td>
<td>( \partial \mathbf{R}_\phi / \partial \rho_p )</td>
<td>( \mathbf{1}_K \mathbf{1}_K^T/K )</td>
</tr>
</tbody>
</table>

Furthermore, FIM elements for FCD sources may be computed as in expression (72), being their components defined as it is shown in (73)-(76).
\[
F_{\Psi} = \sum_{k=1}^{N_s} G_k J_{\bar{\Psi}_1} G_k^T + \sum_{q_1=1}^{2N_c} \sum_{q_2=1}^{2N_c} C_{3}^{(q_1,q_2)} e_N^{(q_1)} e_N^{(q_2)} T + \frac{N_s(K-1)}{\sigma^2_w} e_N^{(4)} e_N^{(4)} T
\] (72)

\[
\bar{\Psi}_1 = [k_0, \lambda_n, \gamma_k, \sigma^2_w]^T G_k = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & K\lambda^{(k)} & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & P_s K \frac{\partial \lambda^{(k)}}{\partial \rho} & 0
\end{bmatrix}
\] (73)

\[
\gamma_k = K\lambda^{(k)} Ps
\] (74)

\[
\{ J_{\bar{\Psi}_1} \}_{pq} = tr \left( R_k^{-1} \frac{\partial R_k}{\partial \bar{\Psi}_1 p} R_k^{-1} \frac{\partial R_k}{\partial \bar{\Psi}_1 q} \right) + \frac{N_s}{\sigma^2_w} \begin{bmatrix}
tr \left( R_k^{-1} \frac{\partial R_k}{\partial \gamma_k} R_k^{-1} \frac{\partial R_k}{\partial \gamma_k} \right)
\end{bmatrix}
\] (75)

\[
C_{3}^{(q_1,q_2)} = \sum_{k=1}^{N_s} \sum_{l=1}^{N_s} \left( \lambda^{(k)} - \lambda^{(l)} \right)^2 \left( u^{(l)} \right)^T \frac{\partial u^{(k)}}{\partial \gamma_{q_1}} \left( u^{(k)} \right)^T \frac{\partial u^{(l)}}{\partial \gamma_{q_2}} + \frac{N_s(K-1)}{\sigma^2_w} \begin{bmatrix}
\end{bmatrix}
\] (76)