Equation system describing asymmetric toggle-switch

Using Table 1 and assuming first and second order reaction kinetics, we find the following non-linear coupled ordinary differential equation representation of the circuit in Fig. 1:

\[
\begin{align*}
\dot{[P]}_2 &= k_{11}[P]^2 - q_{11}[P^2_2] - \left(\frac{\gamma_{1p}}{\sigma_1} - k_{22}[D^2_{00}]\right)[P^1_2] + (q_{22} - k_{24}[P^1_2])[D^2_{10}] + q_{24}[D^2_{20}] \quad (A.1) \\
\dot{[P]}_2 &= k_{21}[P]^2 - q_{21}[P^2_2] - \left(\frac{\gamma_{2p}}{\sigma_2} - k_{12}[D^1_{00}]\right)[P^1_2] + (q_{12} - k_{14}[P^1_2])[D^1_{10}] + q_{14}[D^1_{20}] \quad (A.2) \\
\dot{[P]}_1 &= \alpha_{1p}[M^1] + 2q_{11}[P^1_2] - 2k_{11}[P^1_1]^2 - \gamma_{1p}[P^1] \quad (A.3) \\
\dot{[P]}^2 &= \alpha_{2p}[M^2] + 2q_{21}[P^2_2] - 2k_{21}[P^2_1]^2 - \gamma_{2p}[P^2] \quad (A.4) \\
\dot{[D]}_{00} &= q_{12}[D^1_{10}] + (q_{13} + \alpha_{1m})[D^1_{01}] - (k_{12}[P^2_2] + k_{13}[R])[D^1_{00}] \quad (A.5) \\
\dot{[D]}_{00} &= q_{22}[D^1_{10}] + (q_{23} + \alpha_{2m})[D^1_{01}] - (k_{22}[P^2_2] + k_{23}[R])[D^1_{00}] \quad (A.6) \\
\dot{[D]}_{10} &= k_{12}[D^1_{00}][P^2_2] + q_{14}[D^2_{10}] + (q_{15} + \alpha_{1m})[D^1_{11}] - (q_{12} + k_{14}[P^1_2] + k_{15}[R])[D^1_{10}] \quad (A.7) \\
\dot{[D]}_{10} &= k_{22}[D^1_{00}][P^2_2] + q_{24}[D^2_{10}] + (q_{25} + \alpha_{2m})[D^1_{11}] - (q_{22} + k_{24}[P^1_2] + k_{25}[R])[D^1_{10}] \quad (A.8) \\
\dot{[D]}_{20} &= k_{14}[D^1_{10}][P^2_2] + (q_{17} + \alpha_{1m})[D^1_{21}] - (q_{14} + k_{17}[R])[D^1_{20}] \quad (A.9) \\
\dot{[D]}_{20} &= k_{24}[D^1_{10}][P^2_2] + (q_{27} + \alpha_{2m})[D^2_{21}] - (q_{24} + k_{27}[R])[D^2_{20}] \quad (A.10) \\
\dot{[D]}_{01} &= k_{13}[D^1_{00}][R] - (q_{13} + \alpha_{1m})[D^1_{01}] \quad (A.11) \\
\dot{[D]}_{01} &= k_{23}[D^1_{00}][R] - (q_{23} + \alpha_{2m})[D^1_{01}] \quad (A.12) \\
\dot{[D]}_{11} &= k_{15}[D^1_{10}][R] - (q_{15} + \alpha_{1m})[D^1_{11}] \quad (A.13) \\
\dot{[D]}_{11} &= k_{25}[D^1_{10}][R] - (q_{25} + \alpha_{2m})[D^1_{11}] \quad (A.14) \\
\dot{[D]}_{21} &= k_{17}[D^1_{20}][R] - (q_{17} + \alpha_{1m})[D^1_{21}] \quad (A.15) \\
\dot{[D]}_{21} &= k_{27}[D^1_{20}][R] - (q_{27} + \alpha_{2m})[D^2_{21}] \quad (A.16) \\
\dot{[E]} &= \alpha_{1m}([D^1_{10}] + [D^1_{11}] + [D^1_{21}]) - \gamma_{1m}[E^1] \quad (A.17) \\
\dot{[E]} &= \alpha_{2m}([D^2_{01}] + [D^2_{11}] + [D^2_{21}]) - \gamma_{2m}[E^2] \quad (A.18) \\
\dot{[M]} &= \alpha_{1m}[E^1] - \gamma_{1m}[M^1] \quad (A.19) \\
\dot{[M]} &= \alpha_{2m}[E^2] - \gamma_{2m}[M^2] \quad (A.20) \\
\dot{[R]} &= q_{13}[D^1_{01}] + q_{15}[D^1_{11}] + q_{17}[D^1_{21}] + q_{25}[D^2_{01}] + q_{27}[D^2_{11}] + q_{27}[D^2_{21}] + \alpha_{1m}[E^1] + \alpha_{2m}[E^2] \\
&- (k_{13}[D^1_{00}] + k_{15}[D^1_{10}] + k_{17}[D^1_{20}] + k_{23}[D^2_{00}] + k_{25}[D^2_{10}] + k_{27}[D^2_{20}])[R] \quad (A.21)
\end{align*}
\]
Here, square bracket \([X]\) denotes concentration of the chemical species \(X\), and a dot \(\cdot\) denotes time derivative.