Confounder effect on network reconstruction

In order to study the effect of sequentially removing confounders on the subsequent network reconstruction, we performed a simulation study. We considered 4 different scenarios, in which 2 genes, $Y_1$ and $Y_2$, interact in the presence of a single confounder $X$. In each of the four simulations, the confounder $X_i$ was drawn from i.i.d. $N(0, 1)$, with $i = 1, \ldots, 94$ (as in the DeepSAGE data), whereas the gene expression levels were simulated as follows,

1. Considering a i.i.d. random effects, $\epsilon_i \sim N(0, 1)$, the “connected” genes 1 and 2, were drawn from

   $Y_{i1} \sim \text{Poisson}(\lambda_{i1})$, \hspace{1cm} $\log(\lambda_{i1}) = X_i + \epsilon_i$

   $Y_{i2} \sim \text{Poisson}(\lambda_{i2})$, \hspace{1cm} $\log(\lambda_{i2}) = \epsilon_i$

   Note that only gene 1 is affected by the confounder $X$.

2. The genes 1 and 2 are “unconnected” and drawn from

   $Y_{i1} \sim \text{Poisson}(\lambda_{i1})$, \hspace{1cm} $\log(\lambda_{i1}) = X_i$

   $Y_{i2} \sim \text{Poisson}(\lambda_{i2})$, \hspace{1cm} $\log(\lambda_{i2}) = 0$

   Note that, again, only gene 1 is affected by the confounder $X$.

3. Considering a i.i.d. random effects, $\epsilon_i \sim N(0, 1)$, the “connected” genes 1 and 2, were drawn from

   $Y_{i1} \sim \text{Poisson}(\lambda_{i1})$, \hspace{1cm} $\log(\lambda_{i1}) = X_i + \epsilon_i$

   $Y_{i2} \sim \text{Poisson}(\lambda_{i2})$, \hspace{1cm} $\log(\lambda_{i2}) = X_i + \epsilon_i$

   Note that both genes 1 and 2 are affected by the confounder $X$.

4. The genes 1 and 2 are “unconnected” and drawn from

   $Y_{i1} \sim \text{Poisson}(\lambda_{i1})$, \hspace{1cm} $\log(\lambda_{i1}) = X_i$

   $Y_{i2} \sim \text{Poisson}(\lambda_{i2})$, \hspace{1cm} $\log(\lambda_{i2}) = X_i$

   Note that, again, both genes 1 and 2 are affected by the confounder $X$. 
We performed 1000 iterations for each of the four scenarios and inferred the (partial) correlation coefficient between the genes. As can be seen from the results in Figure 1, the inferred values for the relation between genes 1 and 2 on average exactly correspond to the expected value, i.e., non-zero and zero two times.
Figure 1 Inferred partial correlation coefficient by means of the two-step inference procedure (Poisson regression + Graphical Lasso) between genes 1 and 2 for the four different confounding plus interaction scenarios indicated by the networks above each of the corresponding histograms.