Figure 6: Phase portraits on the sheets $S^r_2$ and $S^r_3$ of the critical manifold $S$ near the fold $F_3$; panel (a) shows a trajectory of the desingularised slow flow (11), which converges to an attracting focus on $F_3$ (grey line); panel (b) shows projected trajectories of the slow flow (9); the repelling and attracting nature of $F_3$ is indicated by light- and dark-green colours, respectively.

obtain

\[
\begin{pmatrix}
\dot{V} \\
\dot{m}_{SO}
\end{pmatrix} = \left( \begin{array}{c}
-\frac{\partial f^*_1}{\partial V}^{-1} \left[ \frac{\partial f^*_1}{\partial m_{SO}} f^*_2 + \frac{\partial f^*_1}{\partial h_{SI}} f^*_3 \right] \\
\frac{\partial f^*_1}{\partial V} f^*_4
\end{array} \right),
\]

where $h_{SI}$ is uniquely determined from $f^*_1(V, m_{SO}, h_{SI}, \lambda) = 0$. We refer to [43] for more details on this step. Note that (10) becomes singular when $\partial f^*_1/\partial V = 0$, that is, precisely where $S$ has folds with respect to $V$. We can desingularise the flow by scaling time with the factor $-\partial f^*_1/\partial V$. This rescaling reverses the direction of the time whenever $\partial f^*_1/\partial V > 0$ and we obtain the desingularised slow flow in the form

\[
\begin{pmatrix}
\dot{V} \\
\dot{m}_{SO} \\
0
\end{pmatrix} = \left( \begin{array}{c}
\frac{\partial f^*_1}{\partial m_{SO}} f^*_2 + \frac{\partial f^*_1}{\partial h_{SI}} f^*_3 \\
\frac{\partial f^*_1}{\partial V} f^*_4
\end{array} \right).
\]

The actual slow flow on $S$ is now defined by the desingularised slow flow (11), where we must take into account the time reversal in the regimes where $\partial f^*_1/\partial V > 0$. Figure 6(a) illustrates this for a neighbourhood of the fold $F_3$ on $S$ that separates the sheets $S^r_2$ and $S^r_3$; we have $\partial f^*_1/\partial V < 0$ on $S^r_3$ and $\partial f^*_1/\partial V > 0$ on $S^r_2$. The phase portraits in Figure 6 are projected onto the $(m_{SO}, V)$-plane. Figure 6(a) shows how a trajectory of (11) near $F_3$ (grey line) is attracted to a focus equilibrium of the desingularised slow flow, marked with a black dot on $F_3$. Figure 6(b) shows the corresponding projection of the slow flow (9) on $S$; note the change in direction of the flow for the region where $\partial f^*_1/\partial V > 0$. The fold $F_3$ in Figure 6(b) is now divided into two parts, a repelling segment on the left side of the focus equilibrium (light-green line) and an attracting segment on the right side of the focus equilibrium (dark-green line). In fact, the focus equilibrium