Figure 1: Responses of system (1) to a current injection of $I_{app} = 20 \mu A/cm^2$ from $t = 50$ to $t = 53$; overlaid are the responses corresponding to different values of the maximal conductance $g_{SI}$ (in mS/cm$^2$) of the slow inward current, namely, $g_{SI} = 0.1$, $g_{SI} = 0.5$ and $g_{SI} = 0.6$, which are examples of responses with no ADP, with ADP and a (three-spike) burst with ADP, respectively.

A short-current injection whose duration guarantees that the rapidly rising membrane potential will reach and cross its local maximum creating a fully developed spike; see [17, 25, 26, 32] for more details. Two of the three typical responses shown in Figure 1 exhibit a positive deflection of the membrane potential characterised by a ‘hump’ in the time trace of the membrane potential at the end of the burst; this is called after-depolarisation (ADP), which can exist, provided $\tau_{mFO} < \tau_{mSI}$ [17]. Only the first response (lower curve) is a spike without ADP. Note that the last trace, which corresponds to $g_{SI} = 0.6$, the highest value of $g_{SI}$ in the example, has sufficiently strong $I_{SI}$ to enable the membrane potential to cross the excitability threshold during the ADP, so that additional spikes are fired.

System (1) defined by equations (2)–(4) evolves on multiple time scales, because $C_m/g_{FO}$ (as an approximation of the time scale for $V$) and $\tau_x$ with $x \in \{ m_{SI}, m_{FO}, m_{SO}, h_{SI} \}$ have different orders of magnitude. As indicated in Table 1, $m_{SO}$ and $h_{SI}$ are slow variables that vary on a time scale that is (roughly) 10 times slower than $m_{SI}$ and $m_{FO}$, and 100 times slower than $V$. In particular, this means that our model is capable of firing an arbitrarily large number of spikes during the ADP. More precisely, an increase in $g_{SI}$, as in Figure 1 and throughout this paper, has the net effect that the slow variable $h_{SI}$ becomes even slower, so that more spikes can be fired during the time it takes for $h_{SI}$ to relax back to its equilibrium value. In this paper we are not interested in the exact nature of this process, but we mention here that a large number...