Appendix 3

Proof of Solution to Computing Clinically Relevant Dynamic Probabilities: Consider the stochastic differential equation \( dX = \mu(X) \, dt + \sigma(X) \, dW \). Given that the current value of \( X(t) \) is \( x \), where \( 0 \leq x \leq b \), let \( u(x) \) denote the probability of reaching the level \( b \). It is shown in [31, p.193] that \( u(x) \) satisfies the nonlinear ordinary differential equation:

\[
\frac{du}{dx} \mu(x) + \frac{d^2u}{dx^2} \frac{\sigma^2(x)}{2} = 0
\]

\( u(0) = 0, \ u(b) = 1 \)

Therefore, in the context of this paper, \( u(x) \) satisfies the following ODE,

\[
\frac{du}{dx} \left[ E_{Ix} - \frac{E_x(x - p_b)}{R} \right] + \frac{d^2u}{dx^2} \frac{\sigma^2E^2x^2}{2} = 0
\]

\( u(0) = 0, \ u(b) = 1 \)

Using Green’s function methods, the solution to the above boundary value problem is given in terms of an important quantity called the scaling function \( S(x) \) [31, p.194-195]:

\[
u(x) = \frac{S(x) - S(0)}{S(b) - S(0)}, \text{ where } S(x) = \int s(\eta) \, d\eta, \text{ and } s(x) = \exp \left[ -\frac{1}{\sigma^2} \int \frac{2\mu(\xi)}{\sigma^2(\xi)} \, d\xi \right].
\]

Identification of the parameters with those of the stochastic Marmarou model,

\[
\mu(p) = \left( \frac{E}{R} \right) p (RI + p_b - p), \ \sigma^2(p) = (\sigma E_p)^2
\]

immediately yields the claimed result in this subsection of the paper.