Additional file 1. Time constants do not affect model equilibria.

The Jacobian matrix of the model can be described as $J = \text{diag}(\tau_1^{-1}, \tau_2^{-1}, \tau_3^{-1})A$, where $A$ is a 3-by-3 matrix, each row of which designates partial derivatives of the right-hand side of the corresponding ODE evaluated at the equilibrium point with respect to state variables. Bifurcation theory states that the Jacobian matrix has a simple zero eigenvalue at a saddle-node bifurcation point $[1\cdot 3]$. That means that the characteristic polynomial of $J$ has a zero degree term (i.e., $\det J$) of zero, and therefore, $\frac{\det A}{\tau_1 \tau_2 \tau_3} = 0$. This equation indicates that the saddle-node bifurcation of the model is independent of the time constants because the equality holds irrespective of them. Thus, the bifurcation can be thoroughly analyzed in a relatively low-dimensional parameter space consisting of $j_1, j_2, j_3,$ and $j_4$. In contrast to the saddle-node bifurcation of the model, its Hopf bifurcation is dependent on the time constants, which means that the higher-dimensional parameter space needs to be explored, making analysis much more difficult. However, despite an extensive search, the model did not exhibit any Hopf bifurcation around the default values of the parameters.

References

