Algorithm-1 Hitting-1($X, \mathcal{T}, w, t$)

**Input:** An instance of the weighted $t$-cover hitting set problem ($t \geq 1$)

**Output:** A minimum weight $t$-cover hitting set

1.1 Sort $\mathcal{T}$ into $\{S_1, S_2, \ldots, S_n\}$ such that $|S_i| \leq |S_j|$ for any $i < j$;
1.2 Compute $X_i = \bigcup_{j=1}^i S_j$, $k_i = |X_i|$ for all $1 \leq i \leq n$;
1.3 Sort $X$ into $\{u_1, u_2, \ldots, u_m\}$ such that for any $1 < i \leq n$,
   if $u_i \in X_{i-1}$, $u_{i+1} \in X_i - X_{i-1}$, then $i_1 < i_2$;
1.4 $H_m = X$;
1.5 $Q_{old} = \{ (hit(\emptyset), \emptyset) \}$; $Q_{new} = \{ (hit(\emptyset), \emptyset) \}$;
2 for $i = 1$ to $m$ do /* test each element in $X$ */
3 for each $P = (hit(H), H) \in Q_{old}$ do
4.1 $P' = (hit(H \cup \{u_i\}), H \cup \{u_i\})$;
4.2 if $hit(H \cup \{u_i\}) = [t, t, \ldots, t]$ then
4.3 if $\text{weight}(H_m) > \text{weight}(H \cup \{u_i\})$ then
4.4 $H_m = H \cup \{u_i\}$;
4.5 else
4.6 if there is no $(hit(H'), H')$ in $Q_{new}$ such that $hit(H') = hit(H \cup \{u_i\})$ then
4.7 $Q_{new} = Q_{new} \cup \{P'\}$; /* $u_i$ hits some additional subsets and new sub-solution is added. */
4.8 else /* $u_i$ does not hit additional subsets but may reduce the total weight */
4.9 find $(hit(H'), H')$ in $Q_{new}$ such that $hit(H') = hit(H \cup \{u_i\})$;
/* When two sub-solutions cover subsets of $\mathcal{T}$ in the same way, keep the sub-solution with smaller weight. */
4.10 if $weight(H') > weight(H \cup \{u_i\})$ then
4.11 replace $(hit(H'), H')$ with $(hit(H'), H \cup \{u_i\})$;
/* make sure that all subsets in $\mathcal{T}_X[1:i]$ are hit by at least $t$ elements in each sub-solution; */
/* this is in order to remove any sub-solution that cannot be extended to a full-solution. */
5.1 remove any $P = (hit(H), H)$ from $Q_{new}$ if any subset in $\mathcal{T}_X[1:i]$ is not hit by $t$ elements in $H$;
5.2 $Q_{old} = Q_{new}$;
6 return $H_m$;