two popular cost functions have been studied. The first is the classical Euclidean distance, given by

\[ \Theta_E(W, H) \equiv (\sum_{j=1}^{n} \|v_j - Wh_j\|_2^2)^{\frac{1}{2}} = \|V - WH\|, \]  

(2)

Another measure usually used in practice is K-L divergence (Kullback-Leibler divergence)

\[ \Theta_D(V||WH) \equiv \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \frac{V_{ij} \log \frac{V_{ij}}{\sum_{l=1}^{r} W_{il} H_{lj}}}{\sum_{i=1}^{m} W_{ii} H_{ii}} - V_{ij} + [WH]_{ij} \right) \]  

(3)

The above measure is known as the generalized Kullback-Leibler (KL) divergence. The NMF’s goal is to minimize the distance between matrix \( V \) and \( WH \). In this paper, we choose Euclidean distance as the objective function of the NMF. Use the following iteration formulas [16], to obtain the matrix \( W \) and \( H \) until the \( \Theta_E(W, H) \) value reaches a local minimum:

\[ H_{aj} \leftarrow H_{aj} \frac{[W^T V]_{aj}}{[W^T WH]_{aj}}, W_{ia} \leftarrow W_{ia} \frac{[VH^T]_{ia}}{[WHHT]_{ia}}; \]  

(4)

B. NMF-NMF-SQ Hashing Algorithm

Figure 1 shows the calculation process of a NMF-NMF-SQ hashing algorithm proposed in [20], described as follows:

1) Given an image, using the private key \( k_1 \) to pseudorandomly select sub-images \( A_i, (A_i \in R^{m \times m}, 1 \leq i \leq p) \).
2) Obtain the NMF of each sub-image:

\[ A_i \approx W_i F_i^T, (W_i, F_i \in R^{m \times r_1}). \]  

(5)

where \( r_1 (r_1 \ll m) \) is the rank. In this way, we get \( 2p \) matrix in size \( m \times r_1 \).