The corresponding difference equation is

$$y_k[n] = x[n] + e^{j \frac{2\pi}{N} y_k[n-1]}, \quad \text{with } y_k[-1] = 0.$$  \hfill (12)

This first order difference equation contains a complex multiplication factor, which is computationally demanding. To save the computational cost, the transmission function can be extended in both the numerator and the denominator by the conjugate of $(1 - e^{j \frac{2\pi}{N} z^{-1}})$, which leads to

$$H_k(z) = \frac{1 - e^{-j \frac{2\pi}{N} z^{-1}}}{(1 - e^{-j \frac{2\pi}{N} z^{-1}})(1 - e^{j \frac{2\pi}{N} z^{-1}})} = \frac{1 - e^{-j \frac{2\pi}{N} z^{-1}}}{1 - 2 \cos \left( \frac{2\pi}{N} \right) z^{-1} + z^{-2}}.$$  \hfill (13)

The respective difference equation of this second order IIR system is

$$y_k[n] = x[n] - x[n-1] e^{-j \frac{2\pi}{N}} + 2 \cos \left( \frac{2\pi k}{N} \right) y_k[n-1] - y_k[n-2]$$  \hfill (15)

with $x[-1] = y[-1] = y[-2] = 0$. Such a structure can be described using the state variables:

$$s[n] = x[n] + 2 \cos \left( \frac{2\pi k}{N} \right) s[n-1] - s[n-2],$$  \hfill (16)

while the output is given by

$$y_k[n] = s[n] - e^{-j \frac{2\pi}{N} s[n-1]}$$  \hfill (17)

and we set $s[-1] = s[-2] = 0$. The signal flow graph representing the system is depicted in Fig. 2.

The state-space description is advantageous because only the output sample $y[N]$ is of interest. The algorithm iterates the real-number-only system (16) for $(N+1)$ times (beginning with the sample with the time index 0; in the last iteration the input sample $x[N]$ is put equal to zero). Only in the last step is the output $y_k[N]$ calculated.