assume that we have a finite discrete signal \( x_{\text{orig}}[i] \), where \( i = 1, \ldots, l \). The DFT of this sequence yields \( l \) complex coefficients in the frequency domain \( (X_{\text{orig}}[j], j = 1, \ldots, l) \). If we insert \( p \) consecutive zeros to get \( n = l + p \) samples \( (X[j], j = 1, \ldots, n) \) and take its inverse DFT, we end up with an oversampled version of the original signal with \( n \) complex samples \( (x[i], i = 1, \ldots, n) \). This oversampled signal is real if Hermitian symmetry (complex conjugate symmetry) is preserved in the frequency domain, e.g., the set \( \Theta \) of \( p \) zeros is centered at \( \frac{\pi}{2} \). For erasure channels, the sparse missing samples are denoted by \( e[i_m] = x[i_m] \), where \( i_m \)s denote the positions of the lost samples; consequently, for \( i \neq i_m \), \( e[i] = 0 \). The Fourier transform of \( e[i] \) (called \( E[j] \), \( j = 1, \ldots, n \)) is known for the syndrome positions \( \Theta \). The remaining values of \( E[j] \) can be found from the following recursion (see Appendix I):

\[
E[r] = -\frac{1}{h_k} \sum_{t=1}^{k} E[r + t] h_{k-t}
\]  

(38)

where \( h_k \)s are the ELP coefficients as defined in (36) and Appendix I, \( r \) is a member of the complement of \( \Theta \), and the index additions are in \( \text{mod}(n) \). After finding \( E[j] \) values, the spectrum of the recovered oversampled signal \( X[j] \) can be found by removing \( E[j] \) from the received signal (see (99) in Appendix I). Hence the original signal can be recovered by removing the inserted zeros at the syndrome positions of \( X[j] \). The above algorithm, called the Error Locator Polynomial (ELP) algorithm, is capable of correcting any combination of erasures. However, if the erasures are bursty, the above algorithm may become unstable. To combat bursty erasures, we can use the Sorted DFT (SDFT) \(^6\) [1], [59], [117], [118] instead of the conventional DFT. The simulation results for block codes with erasure and impulsive noise channels are given in the following two subsections.

1) Simulation Results for Erasure Channels: The simulation results for the ELP decoding implementation for \( n = 32 \), \( p = 16 \), and \( k = 16 \) erasures (a burst of 16 consecutive missing samples from position 1 to 16) are shown in Fig. 13; this figure shows we can have perfect reconstruction up to the capacity of the code (up to the finite computer precision which is above 320 dB; this is also true for Figs. 16 and 18). By capacity we mean the maximum number of erasures that a code is capable of correcting.

Since consecutive sample losses represent the worst case [59], [117], the proposed method works better for random samples. In practice, the error recovery capability of this technique degrades with the increase of the block and/or burst size due to the accumulation of round-off errors. In order to reduce the round-off error, instead of the DFT, a transform based on the SDFT, or Sorted DCT (SDCT) can be used [1], [59], [117]. These types of transformations act as an interleaver to break down the bursty erasures.

2) Simulation Results for Random Impulsive Noise Channel: There are several methods to determine the number, locations, and values of the impulsive noise samples, namely: Modified Berlekamp-Massey for real fields [119], [120], ELP, IMAT, and Constant False Alarm Rate with Recursive Detection Estimation (CFAR-RDE). The Berlekamp-Massey method for real numbers is sensitive to noise and will not be discussed here [119]. The other methods are discussed below.

ELP Method [105]: When the number and positions of the impulsive noise samples are not known, \( h_t \) in (38) is not known for any \( t \); therefore, we assume the maximum possible number of impulsive noise samples per block, i.e., \( k = \lfloor \frac{n-1}{2} \rfloor \) as given in (96) in Appendix I. To solve for \( h_t \), we need to know only \( n - l \) samples of \( E \) in the positions where zeros are added in the encoding procedure. Once the values of \( h_t \) are determined from the pseudo-inverse [105], the number and positions of impulsive noise can be found.

\(^6\)The kernel of SDFT is \( \exp \left( \frac{2\pi}{n} i g \right) \), where \( g \) is relatively prime w.r.t. \( n \); this is equivalent to a sorted version of DFT coefficients according to a \( \text{mod} \) rule, which is a kind of structured interleaving pattern.