Fig. 6: (a) Initial ISOMAP embedding $v_1, v_2, \ldots, v_J$ of the measurement vectors. (b) Initial estimates $\{\hat{\theta}_j\}$ of camera positions after rotating and scaling the $\{v_j\}$. (c) Final camera position estimates after running the manifold lifting algorithm. In (b) and (c), the colored points represent the estimated camera positions (color coded by the true $\theta_H^j$ value), while the blue vectors represent the error with respect to the true (but unknown) camera position.

Our decoder will be presented with the ensemble of the measurement vectors $y_1, y_2, \ldots, y_J$ but will not be given any information about the camera positions (save for an awareness of the two caveats mentioned above) and will be tasked with the challenge of recovering the underlying scene $x$.

B. Manifold Lifting Algorithm

We combine the discussions provided in Secs. VI-A and VI-C to design a manifold lifting algorithm that is specifically tailored to this problem.

1) Initial estimates of satellite positions: The algorithm begins by obtaining a preliminary estimate of the camera positions. To do this, we extract from each $y_j$ the measurements corresponding to the two or three coarsest scales (i.e., $y_{j,s_1}, y_{j,s_2}$, and possibly $y_{j,s_3}$), concatenate these into one vector, and pass the ensemble of such vectors (for all $j \in \{1, 2, \ldots, J\}$) to the ISOMAP algorithm. ISOMAP then delivers an embedding of points $v_1, v_2, \ldots, v_J$ in $\mathbb{R}^2$ that best preserves pairwise geodesic distances compared to the input points; an example ISOMAP embedding is shown in Fig. 6(a). What can be inferred from this embedding are the relative camera positions; a small amount of side information is required to determine the proper scaling, rotation, and (possible) reflection of these points to correctly align them with an absolute coordinate system. Assuming that we know the correct vertical and horizontal reflections, after reflecting these camera positions correctly, we then rotate and scale them to fill the square support of $x$.

2) Iterations: Given the initial estimates $\{\hat{\theta}_j\}$ of our camera positions, we can then define the operators $\{R_{\hat{\theta}_j}\}$ and consequently $\hat{R}$. By concatenating the measurement vectors and measurement matrices, initially only those at the coarsest scale (i.e., $y_{j,s_1}$ across all $j$), we write the overall system of equations as $y = \Phi \hat{R}x + n$ as in Sec. VI-A.