Manifold models arise, for example, in settings where a low-dimensional parameter controls the generation of the signal (see Fig. 2). Assume, for instance, that \( x = x_\theta \in \mathbb{R}^N \) depends on some parameter \( \theta \), which belongs to a \( p \)-dimensional parameter space that we call \( \Theta \). One might imagine photographing some static scene and letting \( \theta \) correspond to the position of the camera: for every value of \( \theta \), there is some \( N \)-pixel image \( x_\theta \) that the camera will see. Supposing that the mapping \( \theta \rightarrow x_\theta \) is well-behaved, then if we consider all possible signals that can be generated by all possible values of \( \theta \), the resulting set \( \mathcal{M} := \{ x_\theta : \theta \in \Theta \} \subset \mathbb{R}^N \) will in general correspond to a nonlinear \( p \)-dimensional surface within \( \mathbb{R}^N \).

When the underlying signal \( x \) is an image, the resulting manifold \( \mathcal{M} \) is called an Image Appearance Manifold (IAM). Recently, several important properties of IAMs have been revealed. For example, if the images \( x_\theta \) contain sharp edges that move as a function of \( \theta \), the IAM is nowhere differentiable with respect to \( \theta \) [12]. This poses difficulties for gradient-based parameter estimation techniques such as Newton’s method because the tangent planes on the manifold (onto which one may wish to project) do not exist. However, it has also been shown that IAMs have a multiscale tangent structure [12, 13] that is accessible through a sequence of regularizations of the image, as shown in Fig. 3. In particular, suppose we define a spatial blurring kernel (such as a lowpass filter) denoted by \( h_s \), where \( s > 0 \) indicates the scale (e.g., the bandwidth or the cutoff frequency) of the filter. Then, although \( \mathcal{M} = \{ x_\theta : \theta \in \Theta \} \) will not be differentiable, the manifold \( \mathcal{M}_s = \{ h_s \ast x_\theta : \theta \in \Theta \} \) of regularized images will be differentiable, where \( \ast \) denotes 2D convolution. Tangent planes do exist on these regularized manifolds \( \mathcal{M}_s \), and as \( s \rightarrow 0 \), the orientation of these tangent planes along a given \( \mathcal{M}_s \) changes more slowly as a function of \( \theta \). In the past, we have used this multiscale tangent structure to implement a coarse-to-fine Newton method for parameter

\[ \mathbb{M} = \mathbb{M}_s = \mathbb{M}_s \]