In order to be even more precise in the computation of \( q \), the following polynomial can be solved

\[
(C - 1) \cdot q^{r + 1} + (C + 1) \cdot q^r - (C + 1) \cdot q + 1 - C = 0.
\]

Although from (16) \( \alpha + 1 \) solutions are possible, we select only the \( q \) that belongs to the interval \((0, 1)\). For \( M \leq N \), (16) is equal to the \( q \) of the EFT [11, 16], which means that the PoD of the EFT and the PoD of the M-EFT are the same for \( M < N \). Consequently, the M-EFT automatically inherits from the EFT the property that it outperforms the other matrix-based MOS techniques in the literature for \( M \leq N \) in the presence of white Gaussian noise as shown in [9].

For the sake of simplicity, let us first assume that \( M_1 = M_2 = \ldots = M_R \). Then we can define global eigenvalues as being

\[
\lambda_{G}^{(r)} = \lambda_{1}^{(1)}, \lambda_{1}^{(2)}, \ldots, \lambda_{R}^{(R+1)}. \tag{17}
\]

Therefore, based on (14), it is straightforward that the noise global eigenvalues also follow an exponential profile, since

\[
E \{ \lambda_{G}^{(1)} \} = E \{ \lambda_{1}^{(1)} \} \cdot (q(\alpha_1, \beta_1) \ldots q(\alpha_R, \beta_R))^{i-1}, \tag{18}
\]

where \( i = 1, \ldots, M_{R+1} \).

In Fig. 1, we exemplify the exponential profile property that it is assumed for noise eigenvalues originated from a Wishart matrix. The exponential profile is valid for the noise eigenvalues and also for the global noise eigenvalues. The exemplified data in Fig. 1 has the model order equal to one, since the first eigenvalue does not fit the exponential profile. Hence, the noise eigenvalue profile gets predicted based on the exponential profile assumption and when a gap is detected on the exponential profile the model order is found.

The product across modes increases the gap between the predicted and the actual eigenvalues as shown in Fig. 1. We compare the gap between the actual eigenvalues and the predicted eigenvalues in the \( r \)-th mode to the gap between the actual global eigenvalues and the predicted global eigenvalues. Here we consider that \( X_0 \) is a rank one tensor, and noise is added according to (5). Then, in this case, \( d = 1 \). For the first gap, we have \( \lambda_1^{(r)} - \tilde{\lambda}_1^{(r)} = 2.4 \cdot 10^2 \), while for the second one, we have \( \lambda_1^{(G)} - \tilde{\lambda}_1^{(G)} = 2.4 \cdot 10^{12} \). Therefore, the break in the profile is easier to detect via global eigenvalues than using only one mode eigenvalues.

Since all tensor dimensions may not be necessarily equal to each other, without loss of generality, let us consider the case in which \( M_1 \geq M_2 \geq \ldots \geq M_{R+1} \). In Figs. 2 and 4 we have sets of eigenvalues obtained from each \( r \)-mode of a tensor with sizes \( M_1 = 13, M_2 = 11, M_3 = 8 \) and \( M_4 = 3 \). The index \( i \) indicates the position of the eigenvalues in each \( r \)-th eigenvalues set.

We start by estimating \( d \) with a certain eigenvalue based model order selection method considering the first unfolding only, which in the example in Fig. 2 has a size \( M_1 = 13 \). If \( d < M_2 \), we could have taken advantage of the second mode as well. Therefore, we compute the global eigenvalues \( \lambda_{G}^{(r)} \) as in (17) for \( 1 \leq i \leq M_2 \), thus discarding the \( M_1 - M_2 \) last eigenvalues of the first mode. We can obtain a new estimate \( d \). As illustrated in Fig. 3 we utilize only the first \( M_2 \) highest eigenvalues of the first and of the second modes to estimate the model order. If \( d < M_3 \) we could continue in the same fashion, by computing the global eigenvalues considering the first 3 modes. In the example in Fig. 4 since the model order is equal to 6, which is greater than \( M_4 \), the sequential definition algorithm of the global eigenvalues stops using the three first modes. Clearly, the full potential of the proposed method can be achieved when all modes are used to compute the global eigenvalues. This happens when \( d < M_{R+1} \), so that \( \lambda_{i}^{(G)} \) can be computed for \( 1 \leq i \leq M_{R+1} \).

Note that by using the global eigenvalues, the assumptions of M-EFT, that the noise eigenvalues can be approximated by an exponential profile, and the assumptions of AIC and MDL, that the noise eigenvalues are constant, still hold. Moreover, the maximum model order is equal to \( \frac{M}{2} \) as in (6) in order to avoid the overfitting issue of the EFT.