The topology determines the span of a manifold with respect to its connectivity and dimensionality. In this work, we suggest a 1D closed-loop structure to represent the identity manifold and there are several important considerations to support this seemingly arbitrary but actually practical choice. First, the learning of a higher-dimensional manifold requires a large set of training samples that may not be available for a specific ATR application where only a relatively small candidate pool of possible targets-of-interest is available. Second, this identity manifold is assumed to be closed rather than open, because all targets in our ATR problem are man-made ground vehicles which share some degree of similarity with extreme disparity unlikely. Third, the 1D closed structure would greatly facilitate the inference process for online ATR tasks. As a result, the manifold topology is reduced to a specific ordering relationship of training targets along the 1D closed identity manifold. Ideally, we want targets of the same class or those with similar shapes to stay closer on the identity manifold compared with dissimilar ones. Thus we introduce a class-constrained shortest-closed-path method to find a unique ordering relationship for the training targets. This method requires a view-independent distance or dissimilarity measure between two targets. For example, we could use the shape dissimilarity between two 3D target models that can be approximated by the accumulated mean square errors of multi-view silhouettes.

Assume we have a set of training silhouettes from \( N \) target types belonging to one of \( Q \) classes imaged under \( M \) different views. Let \( y_{mk} \) denote the vectorized silhouette of target \( k \) under view \( m \) (after the distance transform [7]) and let \( L_k \) denote its class label, \( L_k \in [1,Q] \) (\( Q \) is the number of target classes and each class has multiple target types). Also assume that we have identified a LD identity latent space where the \( k \)'th target is represented by the vector \( z_k, k \in \{1,\cdots,N\} \) (\( N \) is the number of total target types). Let the topology of the manifold spanning the space of \( \{z_k|k=1,\ldots,N\} \) be denoted by \( T = [t_1,t_2,\cdots,t_{N+1}] \) where \( t_i \in [1,N] \), \( t_i \neq t_j \) for \( i \neq j \) with the exception of \( t_1 = t_{N+1} \) to enforce a closed-loop structure. Then the class-constrained shortest-closed-path can be written as

\[
T^* = \arg \min_T \sum_{i=1}^N D(z_i^u,z_i^{v+1}),
\]

where \( D(z_i^u,z_i^v) \) is defined as

\[
D(z_i^u,z_i^v) = \sum_{m=1}^M ||y_{m}^u - y_{m}^v|| + \beta \cdot \epsilon(L_u, L_v),
\]

\[
\epsilon(L_u, L_v) = \begin{cases} 0 & \text{if } L_u = L_v, \\ 1 & \text{otherwise}, \end{cases}
\]

where \( ||.|| \) represents the Euclidean distance and \( \beta \) is a constant. The first term in (2) denotes a view independent shape similarity measure between targets \( u \) and \( v \) as it is averaged over all training views. The second term is a penalty term that ensures targets belonging to the same class to be grouped together. The manifold topology \( T^* \) defined in (1) tends to group targets of similar 3D shapes and/or the same class together, enforcing the best local semantic smoothness along the identity manifold, which is essential for a valid shape interpolation between target types.

It is worth mentioning that the identity manifold to be learned according to \( T^* \) will encompass multiple target classes each of which has several sub-classes. For example, we consider six classes of vehicles in this work each of which includes six sub-class types. Although it is easy to understand the feasibility and necessity of shape interpolation within a class to accommodate in-class variability, the validity of shape interpolation between two different classes may seem less clear. Actually, \( T^* \) not only defines the ordering