These questions did not target specifically the neural oscillator but more generally the whole setup at hand. Hence, for the sake of simplicity in the first subject test, we employed a *linearized* version of the neural oscillator, that is a simple oscillator obtained using the same model structure and applying $b = \alpha = 0$.

We found informally that it was generally easy to increase the amplitude of the oscillation, and it was often relatively easy to speed up the oscillator or slow it down, but it tended to be more difficult to decrease the amplitude or stop the oscillator. For this reason, we decided to study how well a subject could coordinate with the neural oscillator’s motion in such a manner as to stop it, showing evidence of truly sharing control with it in *all* interaction modes. In particular, we focused on the situation in which the oscillator was started from the home position with an initial negative velocity, and the subject was asked to try to stop the output sound in as few oscillation “bounces” as possible. To promote high-fidelity force-feedback interaction, the unloaded natural frequency of the neural oscillator was set to a haptic rate of $\omega = 5.0$ rad/sec, corresponding to about 0.8Hz.

**First Four Models**

We calibrated five different models, for which we planned to later estimate and compare their “intrinsic difficulties” relating to stopping the oscillator. The first four models differed only in the implementation of $k_C$, allowing to adjust how strong the force-feedback link between the force-feedback device and oscillator was. $k_C$ ranged from a small but non-negligible value for *WEAK*, to a medium-sized value for *MED*, to large enough to force the device and oscillator position to remain phase-locked for the *STRNG* “strong” model. Figures 6–8 provide some intuition into how the positions of the force-feedback device and of the neural oscillator influence each other, ranging from the *WEAK* model, to the *MED* “medium” model, to the *STRNG* model. The plots are shown only for subject two, but the coupling affected all of the subjects in the same manner. In the *NF* “no force-feedback” model, $k_C$ had the same value as *MED* except that the force-feedback was disabled.

**Fifth Model NF-HINT**

The fifth model was somewhat different. We included it to study how a visual cue providing a strategy could help the subject perform the task better given weak or non-existent force feedback, where the positions of the force-feedback device and the oscillator might not be well correlated.

In the following analysis, we assumed that the force-feedback device would move according to a decaying sinusoid at $\omega$ rad/sec. Even though no test subject produced this trajectory perfectly, many were similar, and the assumption allowed for a simple analysis that provided important insight into the optimal phase relationship. When force feedback is sufficiently weak (e.g. for the *NF* and *WEAK* models), then because the “spring” force on the neural oscillator is proportional to the difference in between its position and the position of the force-feedback device, the most energy-efficient strategy for stopping the oscillations the fastest is for the test subject to force the device along a position trajectory that lags that of the neural oscillator’s position by $90^\circ$. However, according to the theory of dynamic patterns, a $90^\circ$ visual phase relationship should be difficult for test subjects to maintain because it is considered “unstable” (see Section 2.3) [19,13].

Hence, we designed *NF-HINT* to be the same as the *NF* model, except that, instead of displaying the position of the Large oscillator on screen in yellow, we displayed, in green, a ball that moved in proportion to the negative velocity of the oscillator. Then an energy-optimal solution for the subject would be to perfectly follow the green ball. This $0^\circ$ visual phase relationship should be more stable for the human motor control system, at least for visually dominated coordination tasks. In other words, the motion of the green ball represented

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**Fig. 6** Part of subject two’s performance for *WEAK*

**Fig. 7** Part of subject two’s performance for *MED*

**Fig. 8** Subject two’s performance for *STRNG*