Cumulative Game Tree

A simpler representation of a game tree $G_g$ is its corresponding cumulative game tree (CuGa-Tree), $C_g$. A CuGa-Tree $C_g$ is the tuple $(V, E, l, p, n)$:

$$l : E \rightarrow \mathbb{N}, \quad p : E \rightarrow \mathbb{R}^+, \quad n : E \rightarrow \mathbb{N}$$

A CuGa-Tree summarizes the edges of a node with the with the same probability $p$, i.e., with the same number of decomposition decisions, i.e., with the same number of "1"s in the edge label of the game tree. Thus the edge label of a CuGa-Tree indicates the number of decomposition decisions, i.e., the number of subbands which are further decomposed. We have to keep track how many edges of the game tree are summarized by an edge of a CuGa-Tree, therefore we introduce a weight function $n : E \rightarrow \mathbb{N}$. A CuGa-Tree with depth 2 is shown in figure 9.

The edges and edge labels are determined by the predecessor edge (see figure 9(b)): the successors of an edge with label $l(e) = i$ (this number of subbands have been decomposed) can be in the range of 0 to $4i$, as every subband may have up to four children:

$$l(\text{pre}(e)) = i, \quad l(e) \in \{0, \ldots, 4i\}$$

The probability for an edge is similar to game trees:

$$p(e) = p_i^{l(e)}(1 - p_i)^{4l(\text{pre}(e)) - l(e)}$$

The number of edges in the game tree with the same probability is derived by counting the number of edges with the same number of "1"s, i.e., decomposition decisions in the edge label: There are $4l(\text{pre}(e))$ possible positions for $l(e)$ "1"s and thus there are $\binom{4l(\text{pre}(e))}{l(e)}$ edges with the same probability in the game tree:

$$n(e) = \binom{4l(\text{pre}(e))}{l(e)}$$