where $h(x)$ is given by

$$h(x) = -\frac{1}{135}x \left( -9x^2 + 30(1 + \log(5))x + (50 - 30x) \log(5 - 3x) - 50 \log(5) \right).$$

Note that the exact solution for this problem is $y(x) = 4 - 3x$.

Obviously, $K = x(x-t)^2$ is a positive on $I \times I$ and the functions $w(x) = 0$ and $v(x) = 4$ form, respectively, initial ordered lower and upper solutions of (5.3)-(5.4) on $I$. Further, $f(y) = 1/(1+y)^2$ is a strictly decreasing function with

$$-1 \leq \frac{\partial f}{\partial y} < 0 \text{ on } [w, v].$$

Hence, we choose $\sigma = 1$. The graphs of $s_k$ and $S_k$ for $k = 0, 1, 2, 3$ together with the exact solution $y(x)$ are plotted in Figure 1. Notice that the sequences $\{s_k\}$ and $\{S_k\}$ converge to the exact solution, $y(x)$. To measure the bound of the error (or the approximation error) at each iteration $k$, we use the $L_2$-norm defined as

$$E_U^{(k)} = \|S_k(x) - y(x)\|^2 = \int_0^1 (S_k(x) - y(x))^2 dx,$$

and

$$E_L^{(k)} = \|s_k(x) - y(x)\|^2 = \int_0^1 (s_k(x) - y(x))^2 dx.$$

Table 1 shows that just after three iterations the errors $E_U^{(k)}$ and $E_L^{(k)}$ are of the order $10^{-11}$. 

Figure 1: Graphs of $s_k$, $S_k$ and $y$ ($k = 0, 1, 2, 3$) for Example: $s_k$ (dashed); $S_k$ (solid); $y$ (dotted).