A single path channel ($M = 1$) and a two path channel ($M = 2$) with a small excess delay ($\Delta \tau_2 := \tau_2 - \tau_1 = 0.81T_s$), both with a memory length $L = 10$. The parameters of the channels are given in Tab. I. Furthermore, the maximum relative curvatures $\Gamma^N$ and $\Gamma^T$ for different SNRs and the value of $1/\sqrt{\mathcal{F}_{0.95}^{0.95}_{P,N-p}}$ are listed in Tab. I. It can be concluded that the planar assumption is always fulfilled since $\Gamma^N$ is much smaller than $1/\sqrt{\mathcal{F}_{0.95}^{0.95}_{P,N-p}}$ in all cases. This means the likelihood method is always accurate. For the single path channel the uniform coordinate assumption is also fulfilled for all SNRs (see Tab. I), i.e., the confidence regions based on the linearization method and the approximate covariance matrices are accurate. This is confirmed by Fig. 4(a)-4(c). In Fig. 4 the confidence regions based on the linearization method (black ellipse) and the likelihood method (filled dots) are plotted for the parameter combination of the real part $\theta_1$ and the delay $\theta_3$ of the LOS path normalized with respect to the symbol duration $T_s$. Both regions are similar for the single path channel. In case of the two path channel a different situation is observed as shown in Fig. 4(d)-4(f). The uniform coordinate assumption is violated at low SNR since $\Gamma^T$ is not much smaller than $1/\sqrt{\mathcal{F}_{0.95}^{0.95}_{P,N-p}}$ (see Tab. I). The shape of the likelihood confidence region differs strongly from the ellipse generated by the approximate covariance matrix. Only at high SNR both shapes coincide. For the two path channel the uniform coordinate assumption is valid from approximately 35–40 dB upwards. For different channel realizations different results are obtained. It should be mentioned again that the curvature measures strongly depend on the parameter set $\theta$ and on the noise samples. The larger the excess delay $\Delta \tau_2$, the lower is the nonlinearity of the problem, i.e., the uniform coordinate assumption is already valid at lower SNR and vice versa. It can be summarized that the confidence regions based on the linearization method are not accurate at low SNR in a multipath scenario. Hence, the soft information based on the approximate covariance matrix may lead to inaccurate results. The influence of soft information on positioning is investigated in the following section.

V. POSITIONING

A. Positioning based on the Time of Arrival

There are many different approaches to determine the position, e.g., multilateration, radiolocation, fingerprinting, and motion sensors. This paper focusses on radiolocation based on the TOA, which is also called multilateration. Furthermore, two-dimensional positioning is considered in the following. An extension to three dimensions is straightforward.

The position $\mathbf{p} = [x, y]^T$ of a mobile station (MS) is determined relative to $B$ reference objects (ROs) whose positions $\mathbf{p}_b = [x_b, y_b]^T$ $(1 \leq b \leq B)$ are known. For each RO $b$ the TOA $\hat{\tau}_{1,b}$ is estimated. The TOA corresponds to the distance between this RO and the MS $r_b = \hat{\tau}_{1,b} c$, where $c$ is the speed of light. The estimated distances $\mathbf{r} = [r_1, ..., r_B]^T$ are called pseudo-ranges since they consist of the true distances $\mathbf{d}(\mathbf{p}) = [d_1(\mathbf{p}), ..., d_B(\mathbf{p})]^T$ and estimation errors $\mathbf{\eta} = [\eta_1, ..., \eta_B]^T$ with covariance matrix $\mathbf{C}_\eta = \text{diag}(\sigma_{\eta_1}^2, ..., \sigma_{\eta_B}^2)$:

$$\mathbf{r} = \mathbf{d}(\mathbf{p}) + \mathbf{\eta}. \quad (42)$$

The true distance between the $b$th RO and the MS is a nonlinear function of the position $\mathbf{p}$ given by

$$d_b(\mathbf{p}) = \sqrt{(x - x_b)^2 + (y - y_b)^2}. \quad (43)$$