Fig. 1. Characterization of working points in the achievable rate region (left) and achievable power region (right) for DSM designs (6)-(13). DSM designs (6) that reflect

\[
\begin{align*}
R_1, & \quad \text{target} R_2, \\
R_1 & \quad \text{target} R_2, \\
P_1 & \quad \text{tot} P_2, \\
P_1 & \quad \text{tot} P_2,
\end{align*}
\]

\[\Delta \mathbf{R} = \mathbf{P}_g(\Delta \mathbf{R}) \quad \text{and} \quad \mathbf{P}_f(\Delta \mathbf{R})\]

\[\Delta \mathbf{R} \quad \text{can be decomposed into two dimensions ('price of greening' dimension g and fairness dimension f) given by the projections} \quad \mathbf{P}_g(\Delta \mathbf{R}) \quad \text{and} \quad \mathbf{P}_f(\Delta \mathbf{R})\],

and power reductions \(\Delta \mathbf{P}\) that correspond to a reduction in data rates \(\Delta \mathbf{R}\).

We define two dimensions \(g\) and \(f\) as follows

\[
g = -w \eta, \quad f = v \zeta
\]

with \(\eta\) and \(\zeta\) scalar multiples, and vector \(v\) orthogonal to the rate vector before greening \(\mathbf{R}_o\).

Dimension \(g\) lies along the direction of the weight \(w\) (of (7)) in \(\mathbb{R}^2\), and can be seen as the negative gradient of the rate region, i.e., the direction in which the weighted rate sum decreases the fastest. This dimension indicates the ‘price of greening’.

Dimension \(f\) is orthogonal to the direction in which the data rates are reduced proportionally in \(\mathbb{R}^2\). This dimension gives the direction in which data rates are distributed unfairly, and it thus indicates the ‘fairness’ dimension.

A particular data rate reduction \(\Delta \mathbf{R}\) for given power reductions \(\Delta \mathbf{P}\), i.e., greening vector, can now be decomposed into these two dimensions so as to understand how green and how fair a particular greening policy performs. More specifically, we can define the projection of a particular data rate reduction \(\Delta \mathbf{R}\) on these two dimensions as follows

\[
\mathbf{P}_g(\Delta \mathbf{R}) = \text{the price of greening}, \\
\mathbf{P}_f(\Delta \mathbf{R}) = \text{the fairness of greening},
\]

which are also shown in Figure 2.

These two dimensions are essential in understanding greening. For instance, data rate reduction \(\Delta \mathbf{R}^A\) has a small component along \(g\), and thus corresponds to a small price of greening, i.e., a good power-rate trade-off. In other words, the distance to the boundary of the rate region before greening is small. However, the data rate distribution over the two users is very unfairly, i.e., user 2 reduces its data rate a lot whereas user 1 even increases its data rate. In contrary, data rate reduction \(\Delta \mathbf{R}^B\) has a large price of greening, but a very fair (proportional) distribution of the data rates. In the ideal case, we should have small components along both dimensions.