We conclude that constant $b$ for QFSK and XOR XC. We restrict on $\kappa \geq 1$ for roughly auxiliary constants $b$ reasonably high. The restriction implies that constant $b$ is always greater than one for roughly $\kappa \geq 0.3$. For practical purposes, we restrict on $\kappa \geq 1/2$ because than the minimal distance $\delta_{\min}^2$ is reasonably high.

$$\left|\alpha\right|^2 - 2\alpha \left|\frac{s_{c_A} - s_{c_A}'}{\left|s_{c_B} - s_{c_B}'\right|^2 - \delta_{\min}^2}\right| + \left|\frac{s_{c_A} - s_{c_A}'}{\left|s_{c_B} - s_{c_B}'\right|^2 - \delta_{\min}^2}\right|^2 \geq 0,$$

$$\left|\alpha\right|^2 - 2\alpha \left|\frac{s_{c_A} - s_{c_A}'}{\left|s_{c_B} - s_{c_B}'\right|^2 - \delta_{\min}^2}\right| + \left|\frac{s_{c_A} - s_{c_A}'}{\left|s_{c_B} - s_{c_B}'\right|^2 - \delta_{\min}^2}\right|^2 \geq 0,$$

where auxiliary constants

$$b = \left|\frac{s_{c_A} - s_{c_A}'}{\left|s_{c_B} - s_{c_B}'\right|^2 - \delta_{\min}^2}\right|, \quad b \geq 0$$

and $c = -b^2 + \left|\frac{s_{c_A} - s_{c_A}'}{\left|s_{c_B} - s_{c_B}'\right|^2 - \delta_{\min}^2}\right|$ are not functions of $|\alpha|$. Thus, the condition (28) has a critical $|\alpha|$ which equals to either $b$ if $b \leq 1$ or limiting value 1 if $b \geq 1$. In Fig. 6, we plot the constant $b$ for all indices $c_A \neq c_A', c_B \neq c_B', c_A \oplus c_B \neq c_A' \oplus c_B'$ for QFSK and XOR XC. We conclude that constant $b$ is always greater than one for roughly $\kappa \geq 0.3$. For practical purposes, we restrict on $\kappa \geq 1/2$ because than the minimal distance $\delta_{\min}^2$ is reasonably high. The restriction implies that constant $b \geq 1$ and so the critical $|\alpha| = 1$.

The minimal distance $\delta_{\min}^2$ is always greater than one for roughly $\kappa \geq 0.3$. For practical purposes, we restrict on $\kappa \geq 1/2$ because than the minimal distance $\delta_{\min}^2$ is reasonably high. The restriction implies that constant $b \geq 1$ and so the critical $|\alpha| = 1$.

For practical schemes where $\kappa \geq 1/2$.  

Figure 7: Distances of hierarchical symbols corresponding to different XC symbols for QFSK and the critical parameter value $\alpha = e^{i\phi}$. Minimal value of modulation index fulfilling the UMP condition is $\kappa = 5/6$ (green thick line meets blue thick line).