The AFT frailty model given in 5 can be equivalently expressed in the log-linear model form:

$$\log(T_{ij}) = \beta_0^* + x_{ij}\beta^* + \delta\epsilon_{ij} + \xi_i,$$  \hspace{1cm} (8)

where $\xi_i \sim N(0, \sigma^2)$ is still the random frailty effect and is independent of $\epsilon_{ij}$. $\epsilon_{ij}$, as in a linear regression model, is the random error that models the random deviation of $\log(T_{ij})$ from the linear predictor part (conditional on $\xi_i$), and $\delta$ is a scale parameter for $\epsilon_{ij}$. $\epsilon_{ij}$ is assumed to follow a certain known probability distribution [4], and different distributions of $\epsilon_{ij}$ correspond to different parametric forms for the survival time. For the Weibull frailty model in 4, the random error $\epsilon$ follows the Gumbel distribution [4], which has density function $f(\epsilon) = \exp(\epsilon - e^\epsilon)$. For the Log-logistic frailty model in 6, the random error $\epsilon$ follows the logistic distribution with density function $f(\epsilon) = \frac{e^\epsilon}{(1 + e^\epsilon)^2}$. Using the parameterization in 5, parameters in model 8 can be expressed as $\delta = \frac{1}{\gamma}$ and $\beta^* = -\beta$.

With expression 8, recall that $x = 1$ denotes intervention and $x = 0$ denote control, thus we have:

$$E[\log \frac{T(x = 1)}{T(x = 0)}] = E[\log T(x = 1) - \log T(x = 0)]$$
$$= E[\log T(x = 1)|\xi] - E[\log T(x = 0)|\xi]$$

plug in model 8

$$= E_\xi[\beta_0^* + \beta + \sigma\epsilon + \xi] - E_\xi[\beta_0^* + 0 + \delta\epsilon + \xi]$$
$$= \beta^*$$

Therefore, model 8 has a population-level interpretation in that $\beta^*$ is population average log ratio of clearance times between intervention arm and control arm.