**Constant-energy profiles.**

In Fig. S1, we present additional INS data, obtained as two-dimensional cuts through the four-dimensional TOF dataset along different directions that are not parallel to the equatorial scattering plane. The cuts in Fig. S1a were taken at the resonance energy ($\hbar \omega = 0.5 \text{ meV}$) and below it ($\hbar \omega = 0.3 \text{ meV}$) in the AFM state along the plane orthogonal to the (111) direction and passing through the R point. The resonant mode can be seen in the center of the panel, surrounded by a hexagonal arrangement of spin-wave branches emanating from six equivalent S points as clarified in Fig. S1b. Two parallel cross-sections through the zone center, taken at the same energies, are shown in Fig. S1c. Here, one sees the intense FM mode at low energies along with the spin-wave intensity appearing as sharp but much weaker intensity spots in a hexagonal arrangement. The location of the cut in the Brillouin zone is explained by Fig. S1d.

The cross-section presented in Fig. S1e shows data in both AFM and AFQ states along the face of the cubic Brillouin zone, which is orthogonal to the (100) direction, taken at the resonance energy ($\hbar \omega = 0.5 \text{ meV}$). In contrast to the diagonal of the Brillouin zone (Fig. 1c), where the two resonant modes appear to be connected by weaker streaks of intensity forming a continuous elliptical feature around the M point, here one can clearly see that the “ellipse” is disconnected from the resonance (this is best seen in the $T = 2.6 \text{ K}$ data). Its intensity is maximized in between the two S points and is therefore likely stemming from the upper part of the spin-wave excitation branch that connects these points and lies in approximately the same energy range, as one can observe from the fit in Fig. 3a.

**Energy-momentum profiles.**

On the next page, in Fig. S2, we also present an extended version of the energy-momentum profiles along all high-symmetry directions of the Q-space, extracted from the $T = 1.5 \text{ K}$ (top) and $T = 2.6 \text{ K}$ (bottom) datasets. Here, both the dispersion of the collective magnon modes (AFM state) and the intensity modulation of the quasielastic response (AFQ state) can be clearly seen over the broad range of momenta. Note the existence of additional weak temperature-dependent intensity at higher energies ($\hbar \omega > 0.8 \text{ meV}$), in particular near the X and $\Gamma$ points.

**Fitting model.**

For the empirical fitting of the magnon energy ($\hbar \omega_{\text{res}}$) in the AFM state and the quasielastic line width ($\Gamma_0$) in the AFQ state in 3-dimensional momentum space (Fig. 3), we utilized the conventional Fourier expansion, which accounts for the cubic symmetry of the Brillouin zone:

$$ f(Q) = \sum_j A_j \exp[i \mathbf{Q} \cdot \mathbf{R}_j]. $$

**Table S1** | Fitting parameters for Eq. S1 that empirically describes the magnon dispersion in the AFM state ($\hbar \omega_{\text{res}}$) and the quasielastic line width in the AFQ state ($\Gamma_0$). All values are given in meV.
Figure S2 | Extended energy-momentum profiles along a polygonal path through high-symmetry directions in \(Q\)-space. a, Collective modes in the AFM state \((T = 1.5 \text{K})\). b, Quasielastic intensity distribution in the AFQ state \((T = 2.6 \text{K})\). The leftmost \(\Gamma\) point corresponds to the direct beam \((Q = 0)\).

Here \(Q = (Q_x, Q_y, Q_z) = 2\pi/a(H, K, L)\) denotes a vector in the reciprocal space, the summation runs over all lattice vectors \(R_j = a(n_x, n_y, n_z)\), and \(A_{n_j}\) are the free fitting parameters. The series is truncated by considering only higher-order terms with \(n_j = n_x^2 + n_y^2 + n_z^2 \leq 16\), resulting in the following expression for the fitting function:

\[
f(Q) \approx A_0 + 2A_1 \cos(2\pi H) + 2A_2 \cos(2\pi K) + 2A_3 \cos(2\pi L) + 2A_4 \cos(2\pi H) \cos(2\pi K) + 2A_5 \cos(2\pi H) \cos(2\pi L) + 2A_6 \cos(2\pi K) \cos(2\pi L) + 2A_7 \cos(2\pi H) \cos(2\pi K) \cos(2\pi L) + 2A_8 \cos(2\pi H) \cos(4\pi K) + 2A_9 \cos(2\pi K) \cos(4\pi L) + 2A_{10} \cos(2\pi H) \cos(2\pi K) \cos(4\pi L) + 2A_{11} \cos(2\pi H) \cos(2\pi L) \cos(4\pi K) + 2A_{12} \cos(2\pi K) \cos(2\pi L) \cos(4\pi K) + 2A_{13} \cos(2\pi H) \cos(4\pi K) \cos(4\pi L) + 2A_{14} \cos(2\pi K) \cos(4\pi K) \cos(4\pi L) + 2A_{15} \cos(2\pi L) \cos(4\pi K) \cos(4\pi L) + 2A_{16} \cos(8\pi H) + 2A_{17} \cos(8\pi K) + 2A_{18} \cos(8\pi L) + 2A_{19} \cos(8\pi H) \cos(4\pi K) + 2A_{20} \cos(8\pi K) \cos(4\pi L) + 2A_{21} \cos(8\pi L) \cos(4\pi K) + 2A_{22} \cos(8\pi H) \cos(8\pi K) + 2A_{23} \cos(8\pi H) \cos(8\pi L) + 2A_{24} \cos(8\pi K) \cos(8\pi L) + 2A_{25} \cos(16\pi H) + 2A_{26} \cos(16\pi K) + 2A_{27} \cos(16\pi L) + 2A_{28} \cos(16\pi H) \cos(4\pi K) + 2A_{29} \cos(16\pi K) \cos(4\pi L) + 2A_{30} \cos(16\pi L) \cos(4\pi K) + 2A_{31} \cos(16\pi H) \cos(8\pi K) + 2A_{32} \cos(16\pi H) \cos(8\pi L) + 2A_{33} \cos(16\pi K) \cos(8\pi L) + 2A_{34} \cos(16\pi H) \cos(16\pi K) + 2A_{35} \cos(16\pi H) \cos(16\pi L) + 2A_{36} \cos(16\pi K) \cos(16\pi L) + 2A_{37} \cos(32\pi H) + 2A_{38} \cos(32\pi K) + 2A_{39} \cos(32\pi L) + 2A_{40} \cos(32\pi H) \cos(4\pi K) + 2A_{41} \cos(32\pi K) \cos(4\pi L) + 2A_{42} \cos(32\pi L) \cos(4\pi K) + 2A_{43} \cos(32\pi H) \cos(8\pi K) + 2A_{44} \cos(32\pi H) \cos(8\pi L) + 2A_{45} \cos(32\pi K) \cos(8\pi L) + 2A_{46} \cos(32\pi H) \cos(16\pi K) + 2A_{47} \cos(32\pi H) \cos(16\pi L) + 2A_{48} \cos(32\pi K) \cos(16\pi L)\]

(S1)

This function has been used to describe both the magnon dispersion at \(T = 1.5 \text{K}\) and the momentum dependence of the quasielastic line width at \(T = 2.6 \text{K}\). The corresponding fitting parameters, listed in Table S1, were obtained by fitting the experimental values of \(\omega_{\text{ex}}\) and \(I_0\) in the whole equatorial \((HHL)\) plane (around 530 points per temperature), which are shown in the upper half of Fig. 3e. The resulting fitting functions are given in the bottom part of Fig. 3e and as black lines in Fig. 3a and Fig. 3c. Despite the empirical nature of this model, our parametrization is certainly commendable for future theoretical efforts.
Energy width of the magnon peak.

From the color maps presented in Fig. 3a, one can see that the width of the magnon peak in energy does not change considerably between the $\Gamma$ and $R$ points. To show this more quantitatively, we have plotted in Fig. S3 the momentum dependence of the peak width and the energy-integrated peak intensity along the $\Gamma'(001)-R(1\frac{1}{2}1\frac{1}{2})-\Gamma''(110)$ line. One can see that the peak width is essentially the same at the $R$ or $\Gamma'$ points as at the intermediate points ($H = 0.25$ or $0.75$), where the intensity is minimized. This proves that the lower intensity at these points is not a result of peak broadening. The apparent increase of the width near $H = 0.1$ and $0.9$ in Fig. S3a is an artifact due to the finite momentum integration window of 0.05 r.l.u. used in our data analysis. Because these regions correspond to the steepest magnon dispersion, momentum integration results in an overestimation of the peak width.

Furthermore, Fig. S3b clearly confirms that the resonance at the $R$ point is indeed localized in momentum-energy space even if one considers the total energy-integrated spectral weight. It therefore represents a local maximum of intensity along all directions along either energy or momentum, as can be also seen in Fig. 2d or 3a. This lets us consider the $R$-point resonance as a separate feature in the spectrum despite its notable hybridization with the neighboring excitations.

Figure S3 | Energy width and energy-integrated spectral weight of the magnon peak. a, $Q$-dependence of the peak width along the $\Gamma'(001)-R(1\frac{1}{2}1\frac{1}{2})-\Gamma''(110)$ direction. b, $Q$-dependence of the energy-integrated peak intensity along the same direction.