Bilateral Monopoly, Identical Distributors, and
Game-Theoretic Analyses of Distribution Channels

VIII. Technical Appendix

In this Technical Appendix we derive our demand system from the value function of a representative consumer. (This is essential to be able to analyze the Competitive-Substitutability Hypothesis.) Then we utilize our original demand system (A4) to report Channel Performance for any number of competing distributors N. The heterogeneous-competitors model, the identical-competitors model, the non-competing, heterogeneous-distributors model, the non-competing, identical-distributors model, and the single-outlet model are embedded in it. We also provide rich details for the non-competing, heterogeneous-distributors model that complement the heterogeneous-competitors model reported in the body of the text.

VIII.A Deriving the Linear-Demand System

We assume that each distributor faces a downward-sloping, linear demand curve derived from the utility function of a representative consumer:

$$U \equiv \sum_{k=1}^{N} (A_k Q_k - B Q_k^2 / 2) - T \sum_{k=1}^{N} \sum_{m=1, m \neq k}^{N} Q_m$$

The $Q_k$ terms denote quantities purchased from the $k^{th}$ distributor. Inter-distributor interaction is modeled as $T \geq 0$. Utility increases at a decreasing rate provided $A_k, B > 0$. Marginal utility is defined as $M U_k = A_k - B Q_k - T \Sigma Q_m, m \in (1, N), m > k$. Only function (1), or a monotonic transformation of it, is compatible with linear-demand.

The representative consumer’s value function is:

$$V \equiv U + Y - \sum_{k=1}^{N} p_k Q_k$$

Price at the $k^{th}$ distributor is $p_k$; $Y$ is total income. Income not spent on the focal product is devoted to expenditures on all other goods (Häckner 2000). Second-order conditions for value maximization require $B > T$. Maximization of (2) reveals that when there are $N$ distributors, demand for the $k^{th}$ distributor is:
\[
Q_k = \left( \frac{(B + \{N - 2\}T)A_k - T \sum_{m=1}^{N} A_m}{(B - T)(B + \{N - 1\}T)} \right) - (B + \{N - 2\}T)p_k + T \sum_{m=1}^{N} p_m
\]

This can be re-written more conventionally as:

\[
Q_k \equiv A_k - bp_k + \theta \sum_{m=1}^{N} p_m.
\]

\[
A_k = \left( \frac{(B + \{N - 2\}T)A_k - T \sum_{m=1}^{N} A_m}{(B - T)(B + \{N - 1\}T)} \right)
\]

where: \( b \equiv \frac{(B + \{N - 2\}T)}{(B - T)(B + \{N - 1\}T)} \), and \( \theta \equiv \frac{T}{(B - T)(B + \{N - 1\}T)} \).

VIII.B Channel Performance with N Competing Distribution Outlets: the Vertically-Integrated System

We begin with the vertically-integrated system as it establishes Channel Performance values for any coordinating wholesale-price policy. We do not report the N-dimensional version of the channel-coordinating menu because we do not know if channel coordination is always attainable, because there may be no set of fixed fees that prevent at least one distributor from non-participation or defection. We offer details in Table A1 for N heterogeneous competitors and identical competitors. Non-competing outlets may be found by setting \( \theta = 0 \), while the single-outlet case requires setting \( \theta = 0 \) and \( N = 1 \).
Table A1  A Vertically-Integrated System’s Performance with N Heterogeneous, Competing Retail Outlets

<table>
<thead>
<tr>
<th>Heterogeneous-Competitors</th>
<th>Identical-Competitors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_i = \frac{b - (N-1)\theta q_{i, vr} + \theta \sum_{k=1}^{N} q_{k, vr}}{(b + \theta)[b - (N-1)\theta]}$</td>
<td>$q_{vr}^{*}$</td>
</tr>
<tr>
<td>$q_{i} = q_{i, vr}$</td>
<td>$q_{vr}^{*}$</td>
</tr>
<tr>
<td>$\sum_{i=1}^{N} q_{i} = \sum_{k=1}^{N} q_{k, vr}$</td>
<td>$Nq_{vr}^{*}$</td>
</tr>
<tr>
<td>$\Pi^{vr} = \frac{[b - (N-1)\theta \sum_{i=1}^{N} (q_{i, vr})^2 + \theta \left(\sum_{i=1}^{N} q_{i, vr}\right)^2}{(b + \theta)[b - (N-1)\theta]} - \sum_{i=1}^{N} f_{i} - F \frac{N(q_{vr}^{*})^2}{[b - (N-1)\theta]} - NF - F$</td>
<td>$\Pi^{vr}$</td>
</tr>
</tbody>
</table>

Note: $q_{i, vr}^{*} \equiv [A_i - (b - \theta)(c + C)] / 2$

VIII.C  N Competing Distribution Outlets: the Naïve Stackelberg Channel

In Table A2 we report the Channel Performance for a manufacturer that serves N distributors with naïve Stackelberg leadership. To save space, we only report the heterogeneous-competitors case; identical-competitors may be found by equating all $Q_{i, vr}^{*}$, $i \in (1, N)$. We do not detail the N-dimensional version of a sophisticated Stackelberg tariff because it also has $(2^N - 1)$ Zones. Resolution of this inordinately complex problem awaits future research.
Table A2  An Uncoordinated, Decentralized Channel’s Performance with N Heterogeneous, Competing Retailers

\[ \hat{Q}_i = b \left( 2N \left[ 2b - (N - 1)\theta \right] Q_{i}^{\nu} - [2b - 2(N - 1)\theta] \sum_{i=1}^{N} Q_{i}^{\nu} \right) / N(2b + \theta) \left[ 2b - (N - 1)\theta \right] \]

\[ \sum_{i=1}^{N} \hat{Q}_i = \left( b \sum_{i=1}^{N} Q_{i}^{\nu} \right) / [2b - (N - 1)\theta] \]

\[ \hat{M} = \left( \sum_{i=1}^{N} Q_{i}^{\nu} \right) / N[b - (N - 1)\theta] \]

\[ \hat{m}_i = \hat{Q}_i / b \]

\[ \hat{\mu}_i = \frac{2N[b - (N - 1)\theta][2b - (N - 1)\theta] Q_{i}^{\nu} + (2b^2 + (2N + 1)\theta - 2N(N - 1)\theta^2) \sum_{i=1}^{N} Q_{i}^{\nu}}{N(2b + \theta)[b - (N - 1)\theta][2b - (N - 1)\theta]} \]

\[ \hat{\pi}_i = \left( \hat{Q}_i^2 / b \right) - f_i - \phi \]

\[ \sum_{i=1}^{N} \hat{\pi}_i = \left( \left( \sum_{i=1}^{N} \left( \hat{Q}_i^2 / b \right) \right) \right) - \sum_{i=1}^{N} f_i - N\phi \]

\[ \phi = \left( \hat{Q}_N^2 / b \right) - f_N \]

\[ \hat{\Pi} = \left( b \sum_{i=1}^{N} \left( Q_{i}^{\nu} \right)^2 \right) / N[b - (N - 1)\theta][2b - (N - 1)\theta] \]

\[ \hat{\Pi}_c = \left[ \frac{b \left[ 2b - (N - 1)\theta \right] \left[ 4N[b - (N - 1)\theta][2b - (N - 1)\theta] + (2b + \theta)^2 \right] \sum_{i=1}^{N} \left( Q_{i}^{\nu} \right)^2 \right]}{N(2b + \theta)^2 \left[ b - (N - 1)\theta \right] \left[ 2b - (N - 1)\theta \right]^2} - \sum_{i=1}^{N} f_i - F \]

Note: \( Q_{i}^{\nu} = [A_i - (b - \theta)(c + C)] / 2 \)

**VIII.D  Heterogeneous, Non-Competing Retailers**

In this Section we illustrate the case of heterogeneous, non-competing retailers.

**VIII.D.1 Heterogeneous Retailers: a Vertically-Integrated System**

Let there be an arbitrary number (N) of outlets, where N is to be determined. Each dyad (denoted by subscript \( k, \ k \in (1, N) \)) sells a unique number of units (\( q_{k}^{*} = \{\alpha_k - \beta(c_k + C)\} / 2 \) ) at a
unique margin ($\mu_k^* = q_k^*/\beta$). The vertically-integrated system (VIS) earns profit:

$$\Pi_k^{VIS} = \sum_{k=1}^{N} \{\mu_k^* q_k^* - f_k^*\} - F = \sum_{k=1}^{N} \{R_k^* - f_k^*\} - F = \sum_{k=1}^{N} \{\pi_k^*\} - F$$  \hspace{1cm} (6)

We rank profits as $\pi_1^* > \pi_2^* > ... > \pi_N^*$. It is profitable to open an outlet if and only if it covers its fixed cost. The optimal number of outlets meets the channel-breadth condition:

$$(R_1^* - f_1^*) \geq (R_2^* - f_2^*) \geq ... \geq (R_n^* - f_n^*) \geq 0 > (R_{n+1}^* - f_{n+1}^*) \geq ... \geq (R_N^* - f_N^*)$$ \hspace{1cm} (7)

To illustrate the implications of our approach we use numerical analysis by employing an augmented version of demand system (9) in the text that is due to Murphy (1977):

$$q_k = (\alpha_k - \beta p_k), \hspace{0.5cm} k \in (1,N)$$

s.t., $\alpha_k \equiv (\alpha - k \cdot a)$,  \hspace{1cm} (8)

where “k” denotes the kth distributor. The first distributor has the largest base level of demand ($\alpha - a$), the second distributor has the second largest demand ($\alpha - 2 \cdot a$), and so forth.

We illustrate this demand curve in Tables A3 and A4 where we set $\alpha = 101$, $a = 1$, and $\beta = 1$, so that $\alpha_1 = 100$, $\alpha_2 = 99$,.... The bilateral-monopoly model is embedded as the special case of $N = 1$. To concentrate on demand differences, we equalize distribution costs across outlets ($c_k = $10, $k \in (1,N)$). We equalize distributors’ fixed costs at one of two possible levels: $f_k = 0$ or $100$ and we set the manufacturer’s costs at $C = $10 and $F = $500. These assumptions generate the results recorded in Tables A3 and A4.

**Tables A3 and A4 about Here**

At this time we focus on the first column of these Tables (the vertically-integrated system); we will examine the other columns later. Comparing Tables A3 and A4 reveals that a VIS opens 80 retail outlets when each outlet’s fixed cost are zero, but only 61 when each outlet’s fixed cost is $100—fixed distribution cost affects the breadth of a vertically-integrated system.

**VIII.D.2 Heterogeneous Retailers: a Coordinated, Decentralized Channel**

Under comparable treatment the coordination of a channel of N non-competitors requires that the per-unit wholesale price equal the manufacturer’s per-unit production cost (one can see this formally from equations (19) in the text by setting $\theta = 0$). Thus there is no need for a menu; it is sufficient to charge a common two-part tariff $\{C, \hat{\phi}_c\}$. The common, non-zero fixed fee $\hat{\phi}_c$
has three interesting effects. First, the \( k \)th retailer participates in the channel if and only if its net revenue covers its fixed cost plus the fixed fee (\( R_k > (f_k + \hat{f}_c) \)). Thus the larger is the fixed fee, the fewer are the retailers who participate in the channel. Second, the least profitable retailer breaks even while the remaining retailers earn positive profits. Third, manufacturer revenue comes solely from the fixed fee (\( \Pi = \hat{n}_c(\hat{f}_c)\hat{f}_c - F \)). Manufacturer profit is maximized (subject to the coordination constraint) when the percentage change in the number of channel participants equals the percentage change in the fixed fee (Ingene and Parry 1995a).

The third columns of Tables A3 and A4 reveal that the channel-coordinating wholesale price ($10) is unaffected by the retailers’ fixed costs. However, the fixed fee that can optimally be extracted from them declines (by $45) in response to a rise in \( f_i \) from $0 to $100. Also, two fewer retailers participate in the channel at the higher fixed cost level.

**VIII.D.3 Heterogeneous Retailers: a Manufacturer Profit-Maximizing, Channel**

By charging a positive per-unit margin, the manufacturer can earn money on each unit sold as well as gain revenues from the fixed fee; however, setting \( W > C \) lowers the net revenue earned by each retailer, so the marginal retailer (and perhaps others) will withdraw from the channel. If the manufacturer were to sell at a loss (\( W < C \)), all retailers would make more money and the manufacturer could set a higher fixed fee (and might sell to more distributors). Thus there is a profit tradeoff between the wholesale price and the fixed fee. The message can be expressed succinctly, although the mathematics is complex (see Ingene and Parry (1995a, 2004) for details). Let there be \( \hat{n} \) retailers. If the least profitable retailer’s share of unit sales (\( S = q_a / \Sigma q \)) is less than (greater than) its “fair share” (1/\( \hat{n} \)), the manufacturer should charge a positive (a negative) per-unit markup.

The last column of Tables A3 and A4 illustrates that the manufacturer’s profit-maximizing (sophisticated Stackelberg) wholesale price falls in response to an increase in the retailers’ fixed costs while the fixed fee that can optimally be extracted from them rises (by $9.50). Once again, fewer retailers participate in the channel at the higher fixed cost level.

**VIII.D.4 Heterogeneous Retailers: the Channel-Breadth Hypothesis**

Figure A1 shows that the importance of fixed distribution costs is independent of inter-distributor competition. Given the parametric values used earlier, the manufacturer should apply the following decision rule:
• If $f_i \leq 912.50$, only the $i^{th}$ retailer should be employed;

• If $912.503 \leq f_i \leq 3,631.25$, both retailers should be employed and a non-coordinating, sophisticated Stackelberg two-part tariff should be used; and

• If $f_i \geq 3,631.25$, only the $j^{th}$ retailer should be employed.
Table A3  Numerical Illustration of a Heterogeneous-Retailers Model with $f_k = 0$

<table>
<thead>
<tr>
<th>Performance Variables</th>
<th>Vertically-Integrated System (VIS)</th>
<th>Uncoordinated</th>
<th>Coordinated</th>
<th>Mfr. Profit-Maximizing</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>80</td>
<td>54</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td>W</td>
<td>NA</td>
<td>$36.75$</td>
<td>$10.00$</td>
<td>$25.50$</td>
</tr>
<tr>
<td>( \phi )</td>
<td>NA</td>
<td>$0.02$</td>
<td>$729.00$</td>
<td>$280.56$</td>
</tr>
<tr>
<td>( \Pi_c )</td>
<td>$42,970.00$</td>
<td>$31,759.66$</td>
<td>$30,210.25$</td>
<td>$31,542.00$</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>NA</td>
<td>$18,821.03$</td>
<td>$19,183.00$</td>
<td>$20,630.00$</td>
</tr>
<tr>
<td>( \pi_i )</td>
<td>$1,600.00$</td>
<td>$708.88$</td>
<td>$871.00$</td>
<td>$759.50$</td>
</tr>
<tr>
<td>( \pi_n )</td>
<td>$0.25$</td>
<td>$0.00$</td>
<td>$0.00$</td>
<td>$0.00$</td>
</tr>
<tr>
<td>( \Sigma \pi )</td>
<td>NA</td>
<td>$12,938.63$</td>
<td>$11,027.25$</td>
<td>$10,912.00$</td>
</tr>
<tr>
<td>( q_i )</td>
<td>40.00</td>
<td>26.625</td>
<td>40.00</td>
<td>32.25</td>
</tr>
<tr>
<td>( q_n )</td>
<td>0.50</td>
<td>0.125</td>
<td>27.00</td>
<td>16.75</td>
</tr>
<tr>
<td>( \Sigma q )</td>
<td>1,620.0</td>
<td>722.25</td>
<td>904.5</td>
<td>784.00</td>
</tr>
<tr>
<td>( \mu_i )</td>
<td>$40.00$</td>
<td>$53.38$</td>
<td>$40.00$</td>
<td>$32.25$</td>
</tr>
<tr>
<td>( \mu_n )</td>
<td>$0.50$</td>
<td>$26.88$</td>
<td>$27.00$</td>
<td>$16.75$</td>
</tr>
</tbody>
</table>

**Note:** These results assume \( \alpha = (101 - k) \), \( \beta = 1 \), \( c_i = C = \$10 \), \( F = \$500 \); furthermore, in this Table we set \( f_i = 0 \).

**Legend:**

- \( n \equiv \text{Number of Retailers} \)
- \( \Pi_c \equiv \text{Channel Profit} \)
- \( \Sigma \pi \equiv \text{Total Retail Profits} \)
- \( q_n \equiv \text{Unit Sales} \)
- \( W \equiv \text{Wholesale Price} \)
- \( \Pi \equiv \text{Manufacturer Profit} \)
- \( k \equiv l \rightarrow \text{Most-Profitable Retailer} \)
- \( \Sigma q \equiv \text{Total Unit Sales} \)
- \( \phi \equiv \text{Fixed Fee} \)
- \( \pi_i \equiv \text{Retail Profit} \)
- \( k \equiv n \rightarrow \text{Least-Profitable Retailer} \)
- \( \mu_i \equiv \text{Channel Margin} \)
Table A4 Numerical Illustration of a Heterogeneous-Retailers Model with $f_k = $100

| Performance Variables | Vertically-Integrated System (VIS) | Decentralized Channel | | | |
|------------------------|-----------------------------------|-----------------------| | | |
| n | 61 | 54 | 25 | 28 | |
| W | NA | $36.75 | $10.00 | $23.50 | |
| $\phi$ | NA | $-99.98$ | $684.00$ | $290.06$ | |
| $\Pi_c$ | $36,252.50$ | $26,359.66$ | $27,600.00$ | $26,836.75$ | |
| $\Pi$ | NA | $13,421.03$ | $16,600.00$ | $17,638.75$ | |
| $\pi_i$ | $1,500.00$ | $708.88$ | $871.00$ | $715.50$ | |
| $\pi_n$ | $0.00$ | $0.00$ | $55.00$ | $0.00$ | |
| $\Sigma \pi$ | NA | $12,938.63$ | $11,000.00$ | $9,198.00$ | |
| $q_i$ | 40.00 | 26.625 | 40.00 | 33.25 | |
| $q_n$ | 10.00 | 0.125 | 28.00 | 19.75 | |
| $\Sigma q$ | 1,525.00 | 722.25 | 850.00 | 742.00 | |
| $\mu_i$ | $40.00$ | $53.38$ | $40.00$ | $33.25$ | |
| $\mu_n$ | $10.00$ | $26.88$ | $28.00$ | $19.75$ | |

Note: These results assume $\alpha_i = (101 - k), \beta = 1, c_i = C = $10, $F = $500; furthermore, in this Table we set $f_k = $100.

1 Values in *italics* are for each retail outlet of the vertically-integrated system.

Legend:

- n = Number of Retailers
- $\Pi_c$ = Channel Profit
- $\Sigma \pi$ = Total Retail Profits
- $q_k$ = Unit Sales
- W = Wholesale Price
- $\Pi$ = Manufacturer Profit
- k = 1 $\rightarrow$ Most-Profitable Retailer
- $\Sigma q$ = Total Unit Sales
- $\phi$ = Fixed Fee
- $\pi_i$ = Retail Profit
- k = n $\rightarrow$ Least-Profitable Retailer
- $\mu_i$ = Channel Margin

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1 Inter-distributor substitutability is modeled as $T > 0$ while independence is modeled as $T = 0$; complementarity would be modeled as $T < 0$.

2 Some results described in this Section were proved by Ingene and Parry (1995a) for a fully general, downward-sloping demand curve.