Appendix 1. Chebyshev collocation method

By defining the non-dimensional variables $\tilde{z} = \frac{z}{H_w/2}$, $\tilde{t} = \frac{t}{H_w/4D}$, $p_d = \frac{p_{ex}(d)}{\rho_w g H_w}$, and $\tilde{u} = \frac{u}{K}$, Eqs. (1)–(5) can be rendered non-dimensional as follows:

$$\tilde{e} \frac{d\tilde{u}}{dt} = \sin \theta - \mu \left[ A \cos \theta - B \right] p_d (\tilde{z} = 1, \tilde{t}) \quad (7)$$

$$\mu = \mu_0 \left[ 1 - a_o \sinh^{-1} \left( \frac{K \tilde{u}}{2u_{ref}} \right) \right] \quad (8)$$

$$\frac{\partial p_d}{\partial t} = \frac{\partial^2 p_d}{\partial \tilde{z}^2} \quad (9)$$

$$p_d = 0 \quad \text{at} \quad \tilde{z} = 1 \quad (10)$$

$$\frac{\partial p_d}{\partial \tilde{z}} = \frac{\psi \tilde{u}}{2} \quad \text{at} \quad \tilde{z} = -1 \quad (11)$$

where $\tilde{e} = \frac{4KD}{g H_w}$, $A = \frac{1 - (1 - \phi)M}{1 + \phi M}$, $B = \frac{M}{1 + \phi M}$, $M = \rho_n H_w / \rho_s H_s$, $\tilde{z} \in [-1, 1]$. Let $p_d(\tilde{z}, \tilde{t})$ be approximately expressed in a series sum of Chebyshev polynomials as follows:

$$p_d(\tilde{z}, \tilde{t}) = p_d^N(\tilde{z}, \tilde{t}) = \sum_{n=0}^{N} a_n(\tilde{t}) T_n(\tilde{z}) \quad (12)$$

where $a_n(\tilde{t})$ are referred to as *discrete Chebyshev coefficients* and $T_n(\tilde{z})$ are the Chebyshev polynomials of the first kind (see Section 2.4 in Canuto et al. 1987). By using the properties of the Chebyshev polynomials $T_n(\tilde{z})$, the partial differentiation of $p_d^N$ with respective to the variable $\tilde{z}$ at the so-called “Gauss-Lobatto” collocation points $\tilde{z}_i = \cos \frac{\pi(i-1)}{N}$, $i = 1, 2, \ldots, N + 1$, can be discretized as

$$\frac{\partial p_d^N}{\partial \tilde{z}} (\tilde{z}_i, \tilde{t}) = \sum_{j=1}^{N+1} R_{i,j} p_d^N (\tilde{z}_j, \tilde{t}) \quad (13-1)$$
\[
\frac{\partial^2 p_d^N}{\partial \bar{z}^2}(\bar{z}_i, \bar{t}) = \sum_{k=1}^{N+1} \sum_{j=1}^{N+1} R_{i,j,k} p_d^N(\bar{z}_k, \bar{t})
\]  

(13-2)

For brevity, we referred the details of the matrix elements \( R_{i,j} \) to Eqs. (53)–(64) (Park and Ryu 2001) or Eqs. (2.4.29) and (3.1.33) (Canuto et al. 1987). The number of the “collocation points”, \( \bar{z}_1, \bar{z}_2, \ldots, \bar{z}_{N+1} \), is \( N+1 \). According to Eq. (10), we set

\[
p_d^N(\bar{z}_i, \bar{t}) = 0.
\]  

(14-1)

By using Eq. (13-1) to discretize the partial differentiation \( \partial p_d / \partial \bar{z} \) in (11), where \( \bar{z}_i = \bar{z}_{N+1} = -1 \), one can obtain

\[
p_d^N(\bar{z}_{N+1}, \bar{t}) = \frac{1}{R_{N+1,N+1}} \left[ \frac{\psi}{2} - \sum_{k=2}^{N} R_{N+1,k} p_d^N(\bar{z}_k, \bar{t}) \right].
\]  

(14-2)

Similarly, one can use Eq. (13-2) to discretize \( \partial^2 p_d / \partial \bar{z}^2 \) in (9) for \( \bar{z}_i = \bar{z}_2, \bar{z}_3, \ldots, \bar{z}_N \) [see Eqs. (3.1.30) and (3.1.33) in Canuto et al. (1987) for details] and obtain

\[
\frac{\partial}{\partial \bar{t}} \left[ \begin{array}{c} p_d^N(\bar{z}_2, \bar{t}) \\ \vdots \\ p_d^N(\bar{z}_N, \bar{t}) \end{array} \right] = \frac{\psi}{2R_{N+1,N+1}} \left[ \begin{array}{c} f_1 \\ \vdots \\ f_{N-1} \end{array} \right]
\]  

(15)

where the elements \( \xi_{i,j} \) and \( f_k \) (\( i, j, k = 1, \ldots, N-1 \)) are the combinations of the \( R_{i,j} \) (referred to the corresponding author for the details of \( \xi_{i,j} \) and \( f_k \)). Next, by using Eq. (14-2) to replace \( p_d(\bar{z} = -1, \bar{t}) \), Eq. (7) can be re-written as

\[
\frac{d\bar{u}}{d\bar{t}} = F_{slid} + \frac{\mu B}{\varepsilon} \left[ \sum_{k=1}^{N-1} \left( \frac{R_{N+1,k+1}}{R_{N+1,N+1}} \right) p_d^N(\bar{z}_{k+1}, \bar{t}) \right] \frac{\psi \bar{u}}{2R_{N+1,N+1}} \left[ \begin{array}{c} f_1 \\ \vdots \\ f_{N-1} \end{array} \right]
\]  

(16)

where \( F_{slid} = \frac{\sin \theta - \mu A \cos \theta}{\varepsilon} \). Eqs. (15) and (16) can be combined as

\[
\frac{d\bar{Y}}{d\bar{t}} = \Omega \bar{Y} + \bar{\zeta}
\]  

(17)
where \( \mathbf{\bar{Y}} = [p_d^N(\bar{z}_2, \bar{t}), p_d^N(\bar{z}_3, \bar{t}), \ldots, p_d^N(\bar{z}_N, \bar{t})]^T \), \( \mathbf{\xi} = [0, 0, \cdots, 0, F_{slide}]^T \), and \( \mathbf{\Omega} \) is a \( N \times N \) matrix that depends upon \( \psi, \mu, \xi, B, \xi_{i, j}, f_k \), and \( R_{i,j} \).

Eq. (17) was solved by using fourth-order Runge-Kutta method (Gear 1971). The solution of \( \bar{Y} \) is obtained through a time stepping-forward process. At any time \( \bar{t}_j \), the solutions of \( \mathbf{\bar{u}}(\bar{t}_j) \) and \( p_d^N(\bar{z}_k, \bar{t}_j) \), \( k = 2, 3, \ldots, N \), are obtained simultaneously without need to use any iteration processes. And then one can substitute \( \mathbf{\bar{u}}(\bar{t}_j) \) and \( p_d^N(\bar{z}_k, \bar{t}_j) \), \( k = 2, 3, \ldots, N \), into Eq. (14-2) to obtain \( p_d^N(\bar{z} = -1, \bar{t}) \). The size of each time step in running the Runge-Kutta scheme was fixed at \( \Delta \bar{t} = 7 \times 10^{-5} \). The displacement \( \bar{x} \) was calculated in the following manner: given the information of \( \mathbf{\bar{u}}(\bar{t}_j) \) and \( \bar{u}(\bar{t}_j) \), one can use Eq. (17) to predict \( \mathbf{\bar{u}}(\bar{t}_{j+1}) \) and thus obtain \( \mathbf{\bar{u}}(\bar{t}_{j+1}) \), and then use the trapezoidal rule to calculate \( \bar{x}_{j+1} = \bar{x}_j + \Delta \bar{x}_j \), where \( \Delta \bar{x}_j \) is approximated by \( \frac{\mathbf{\bar{u}}(\bar{t}_j) + \mathbf{\bar{u}}(\bar{t}_{j+1})}{2} \). Note that

\[
x = \int_0^t u \, dt = \frac{H_2^2 K}{4D} \int_0^t \bar{u} \, d\bar{t} = \frac{H_2^2 K}{4D} \bar{x}.
\]

**Appendix 2. Linear stability analysis**

Let \( \mathbf{\bar{u}} \) and \( p_d \) be decomposed as

\[
\mathbf{\bar{u}} = \mathbf{\bar{u}}^{(o)} + \delta \mathbf{\bar{u}}, \quad p_d = p_d^{(o)} + \delta p_d,
\]

where \( \mathbf{\bar{u}}^{(o)} \) and \( p_d^{(o)} \) denote the basic states of \( \mathbf{\bar{u}} \) and \( p_d \), \( \delta \mathbf{\bar{u}} \) and \( \delta p_d \) denoting small perturbations. By substituting Eq. (19) into (7)–(11) and neglecting higher-order terms, one can obtain

\[
\begin{align*}
\tilde{\mu} \frac{d}{d\bar{t}} \delta \mathbf{\bar{u}} &= \mu^{(o)} \begin{bmatrix} B & \partial p_d \big( \bar{z} = -1, \bar{t} \big) \end{bmatrix} - \\
\partial \mathbf{\bar{u}} &\begin{bmatrix} A & \cos \theta - B \, p_d^{(o)} \big( \bar{z} = -1 \big) \end{bmatrix}
\end{align*}
\]

\[
\frac{\partial}{\partial \bar{t}} \frac{\partial \delta p_d}{\partial \bar{t}} = \frac{\partial^2 \delta p_d}{\partial \bar{z}^2}
\]

\[
\delta p_d = 0 \quad \text{at} \quad \bar{z} = 1
\]

\[
\frac{\partial}{\partial \bar{z}} \frac{\partial \delta p_d}{\partial \bar{z}} = \frac{\psi \delta \mathbf{\bar{u}} \big( \bar{t} \big)}{2} \quad \text{at} \quad \bar{z} = -1
\]

where

\[
\mu^{(o)} = \mu_0 \left[ 1 - a_o \, \sinh^{-1} \left( \frac{K \mathbf{\bar{u}}^{(o)} \big( \bar{z} \big)}{2u_{ref}} \right) \right]
\]

\[
\delta \mu = \frac{\partial \mu^{(o)}}{\partial \mathbf{\bar{u}}^{(o)}} \delta \mathbf{\bar{u}}, \quad \frac{\partial \mu^{(o)}}{\partial \mathbf{\bar{u}}^{(o)}} = \frac{-\mu_o a_o K}{2u_{ref} \left[ 1 + \left( \frac{K \mathbf{\bar{u}}^{(o)} \big( \bar{z} \big)}{2u_{ref}} \right)^2 \right]}.
\]

By using CC method (following Eqs. (12) to (16)), Eqs. (20)-(23) can be discretized as
\[
\frac{d}{dt} \begin{bmatrix}
\delta p_d^N (z_1, \bar{t}) \\
\delta p_d^N (z_2, \bar{t}) \\
\vdots \\
\delta p_d^N (z_N, \bar{t}) \\
\delta \tilde{u} (\bar{t})
\end{bmatrix} = \Pi 
\]

where \( \Pi_{i,j} = \Omega_{i,j} \) for \( i = 1, \ldots, N-1, \ j = 1, \ldots, N \),

\[
\Pi_{N,j} = -\frac{\mu^{(o)} B R_{N+1,j+1}}{\bar{E} R_{N+1,N+1}} \quad \text{for} \quad j = 1, \ldots, N-1, 
\]

\[
\Pi_{N,N} = \frac{\mu^{(o)} B \psi}{2 \bar{E} R_{N+1,N+1}} - \frac{1}{\bar{E}} \frac{\partial \mu^{(o)}}{\partial \tilde{u}^{(o)}} \left[ A \cos \theta - B p_d^{(o)}(z = -1) \right] 
\]

The matrix \( \Omega \) is defined in Eq. (17) and the elements \( R_{i,j} \) defined in (13-1). Let the notation \( \sigma_r \) denote the largest real part of the \( N \) eigenvalues of the \( N \times N \) matrix \( \Pi \). According to the theory of linear dynamical system (Wiggins 2003; Eq. (C1) in Chen et al. 2010), if \( \sigma_r > 0 \) (or \( < 0 \)), the perturbations \( \delta \tilde{u} \) and \( \delta p_d \) will grow (or decay) as time goes by; i.e. the landslide is unstable (or stable). In this study, the eigenvalues were calculated by using an IMSL subroutine.