I. Derivation of high-dynamic pulse-shaped signal model

The modeling process for high-dynamics pulse-shaped signals can be interpreted with a hybrid analog/digital model, as depicted in Figure 1.

The continuous-time baseband data pulse sequence signal \( x(t) \) obtained after digital/pulse sequence conversion can be expressed as

\[
x(t) = \sum_{n=-\infty}^{\infty} x[n] \delta(t-nT_0),
\]

where \( x[n] \) is the digital data sequence and \( T_0 \) is the fixed data cycle period. The PSF filters \( x(t) \) and outputs a non-dynamic pulse-shaped signal \( y(t) \):

\[
y(t) = \sum_{n=-\infty}^{\infty} x[n] h_{PSF}(t-nT_0).
\]

The channel module is then used to simulate the transmission delay characteristics in the high-speed environment. Let the channel module frequency response be \( H_c(j\Omega) \), which can be written as

\[
H_c(j\Omega) = e^{-j\Omega D},
\]
where parameter $D$ is the transmission delay. Therefore, the continuous-time high-dynamics pulse-shaped signal is obtained as

$$y_d(t) = \sum_{n=-\infty}^{\infty} x[n] h_{psf}(t - D - nT_0). \quad (4)$$

The discrete-time high-dynamics pulse-shaped sequence resulting from sampling the continuous-time signal in (4) at time instants $t = kT_s$ ($T_s = 1/f_s$ is the sample period) is given by

$$y_d[k] = \sum_{n=-\infty}^{\infty} x[n] h_{psf}(kT_s - D - nT_0), \quad (5)$$

where $y_d[k] = y_d(kT_s)$.

According to the interpolation theory, basepoint index $m_k$ and fractional interval $\mu_k$ are defined as

$$m_k = \left\lfloor \frac{kT_s - D}{T_0} \right\rfloor, \quad \mu_k = \frac{kT_s - D}{T_0} - m_k, \quad (6)$$

where $\left\lfloor x \right\rfloor$ is the floor operator. Equation (5) can then be rewritten as

$$y_d[k] = \sum_{i=-\infty}^{\infty} x[m_k - i] h_{psf}\left[ (i + \mu_k)T_0 \right], \quad (7)$$

where $i = m_k - n$.

II. Derivation of parameters $m_k$ and $\mu_k$

Transmission delay $D$ can be defined as

$$D = \frac{d(t)}{c}, \quad (8)$$

where $c$ is the speed of light, and $d(t)$ is the signal propagation path length at received time $t$,

expressed as

$$d(t) = d_0 + \int_0^t v(\tau)d\tau, \quad (9)$$

where $d_0$ is the initial distance at $t = 0$, and $v(t)$ is the receiver speed. We assume here that the
transmitter is stationary. In terms of the Doppler effect, the Doppler data rate is defined as

\[ R_d(t) = -\frac{v(t)}{c}R_0, \]  

where \( R_0 = 1/T_0 \) is the transmission data rate. Substituting (10) into (9) gives

\[ d(t) = d_0 - \frac{c}{R_0} \int_0^t R_d(\tau) d\tau, \]  

whose corresponding discrete expression is

\[ d(kT_i) = d_0 - \frac{c}{R_0} \sum_{n=0}^{k} R_d[n]T_i. \]  

In (12), \( R_d[n] = R_d(nT_i) \) is the discrete Doppler data rate. Substituting (12) and (8) into (6) yields

\[ m_k = \left[ \sum_{n=0}^{k} (R_0 + R_d[n])T_i - \frac{d_k}{c}R_0 \right], \]

\[ \mu_k = \sum_{n=0}^{k} (R_0 + R_d[n])T_i - \frac{d_k}{c}R_0 - m_k. \]  

### III. Derivation of equation \( B = HA \)

The impulse response segments are approximated by \( N \)-order polynomials:

\[ h(yi + i_0, \mu'_{i_0}) = P(yi + i_0, \mu'_{i_0}) = \sum_{n=0}^{N} b_n(yi + i_0)\mu'_{i_0}^n, \]  

where \( P(yi + i_0, \mu'_{i_0}) \) are \( N \)-order polynomials, \( b_n(yi + i_0) \) are the polynomial coefficients and are also the coefficients of traditional Farrow structure. We now define the matrix \( B \) of polynomial coefficients as

\[ B = \begin{bmatrix} b_0(-yI) & b_1(-yI) & \cdots & b_N(-yI) \\ b_0(-yI+1) & b_1(-yI+1) & \cdots & b_N(-yI+1) \\ \vdots & \vdots & \ddots & \vdots \\ b_0(yI-1) & b_1(yI-1) & \cdots & b_N(yI-1) \end{bmatrix}. \]  

According to Lagrange interpolation theory, approximating the impulse response segments with \( N \)-order polynomials requires \( N + 1 \) basepoints at each segment. For simplicity, we sample each impulse response segment evenly; the sample points must include the segment end points, to ensure
continuity of the piecewise polynomial. The sample point coordinates are \( (nT/N, h_n(yi + i'_n)) \), where
\[
h_n(yi + i'_n) = h(yi + i'_n, n/N), \quad n = 0, 1, \ldots, N,\]
is the sampled sequence. We now define the sample matrix \( H \) as
\[
H = \begin{pmatrix}
h_0(-yI) & h_1(-yI) & \cdots & h_N(-yI) \\
h_0(-yI + 1) & h_1(-yI + 1) & \cdots & h_N(-yI + 1) \\
\vdots & \vdots & \ddots & \vdots \\
h_0(yI - 1) & h_1(yI - 1) & \cdots & h_N(yI - 1)
\end{pmatrix}
\] (16)

Following Lagrange interpolation theory, the \( N \)-order polynomials \( P(yi + i'_n, \mu'_n) \) can also be written as
\[
P(yi + i'_n, \mu'_n) = \sum_{n=0}^{N} h_n(yi + i'_n) L_n(\mu'_n),
\] (17)
where \( L_n(\mu'_n) \) is defined as
\[
L_n(\mu'_n) = \prod_{j=0, j\neq n}^{N} \frac{\mu'_n - \frac{j}{N}}{\frac{n}{N} - \frac{j}{N}} = \sum_{m=0}^{N} a_n(m) \mu'^{n}.
\] (18)
The coefficients of polynomial \( L_n(\mu'_n) \) in (18) are defined by matrix \( A \) as
\[
A = [a_n(m)]^T,
\] (19)
where \( m \) is the row index, \( n \) is the column index. Table I lists the values of matrix \( A \) when \( N = 1, N = 2, \) and \( N = 3 \).

We define matrix \( L \) to denote polynomial \( L_n(\mu'_n) \)
\[
L = \begin{bmatrix}
L_0(\mu'_n) & L_1(\mu'_n) & \cdots & L_N(\mu'_n)
\end{bmatrix}^T.
\] (20)
and define matrix \( U \) to denote the variable of the polynomial.
\[
U = \begin{bmatrix}
1 & \mu'_1 & \cdots & \mu'^{N}
\end{bmatrix}^T.
\] (21)
Thus, (18) can be rewritten as
\[
L = AU.
\] (22)
Substituting (17) into (14) yields
\[ \mathbf{HL} = \mathbf{BU}. \quad (23) \]

Thus,

\[ \mathbf{B} = \mathbf{HA}. \quad (24) \]

**Table I** Matrix \( \mathbf{A} \) values

<table>
<thead>
<tr>
<th>Polynomial order</th>
<th>Value</th>
</tr>
</thead>
</table>
| \( N = 1 \)      | \[
\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}
\]         |
| \( N = 2 \)      | \[
\begin{bmatrix} 1 & -3 & 2 \\ 0 & 4 & -4 \\ 0 & -1 & 2 \end{bmatrix}
\]         |
| \( N = 3 \)      | \[
\begin{bmatrix} 1 & -\frac{11}{2} & 9 & \frac{9}{2} \\ 0 & 9 & \frac{45}{2} & \frac{27}{2} \\ 0 & -\frac{9}{2} & 18 & \frac{27}{2} \\ 0 & 1 & -\frac{9}{2} & \frac{9}{2} \end{bmatrix}
\]         |

**IV. Comparison of complexities**

The complexities of the Farrow structure and its modifications are listed in Table II.

**Table II** Complexities of the Farrow structure and its modifications

<table>
<thead>
<tr>
<th>Structure</th>
<th>Number of multipliers</th>
<th>Number of adders</th>
<th>Number of coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Farrow structure</td>
<td>((2I + 1)(N + 1) - 1)</td>
<td>(2I(N + 1) - 1)</td>
<td>(2I(N + 1))</td>
</tr>
<tr>
<td>Modified Farrow structure proposed in [5]</td>
<td>((I + 1)(N + 1))</td>
<td>(2I(N + 1))</td>
<td>(I(N + 1))</td>
</tr>
<tr>
<td>Modified Farrow Structure proposed in this paper</td>
<td>((I + 1)(N + 1))</td>
<td>((2I + 1)(N + 1))</td>
<td>(I(N + 1)\gamma)</td>
</tr>
</tbody>
</table>

**V. Detailed simulation results**

We define the PSF as being a raised cosine roll-off filter with a roll-off factor of 0.5; the impulse response of the PSF is truncated with a Hamming window. The detailed properties of the PSFs designed by different methods and parameters are listed in Table III.
<table>
<thead>
<tr>
<th>Method</th>
<th>N</th>
<th>2I</th>
<th>γ</th>
<th>$\delta_p$</th>
<th>$A_i$ (dB)</th>
<th>Number of multipliers</th>
<th>Number of adders</th>
<th>Number of coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagrange</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>$8.2 \times 10^{-2}$</td>
<td>31.5</td>
<td>24</td>
<td>36</td>
<td>18</td>
</tr>
<tr>
<td>$L_2$</td>
<td>5</td>
<td>32</td>
<td>1</td>
<td>$7.5 \times 10^{-4}$</td>
<td>78.2</td>
<td>102</td>
<td>192</td>
<td>96</td>
</tr>
<tr>
<td>Window</td>
<td>5</td>
<td>32</td>
<td>1</td>
<td>$2.3 \times 10^{-4}$</td>
<td>62.0</td>
<td>102</td>
<td>192</td>
<td>96</td>
</tr>
<tr>
<td>Improved Window</td>
<td>5</td>
<td>32</td>
<td>1</td>
<td>$1.1 \times 10^{-4}$</td>
<td>76.4</td>
<td>102</td>
<td>192</td>
<td>96</td>
</tr>
<tr>
<td>Improved Window</td>
<td>5</td>
<td>32</td>
<td>2</td>
<td>$1.2 \times 10^{-4}$</td>
<td>108.8</td>
<td>102</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>Improved Window</td>
<td>5</td>
<td>32</td>
<td>4</td>
<td>$1.2 \times 10^{-4}$</td>
<td>145.3</td>
<td>102</td>
<td>192</td>
<td>384</td>
</tr>
<tr>
<td>Improved Window</td>
<td>5</td>
<td>32</td>
<td>8</td>
<td>$1.2 \times 10^{-4}$</td>
<td>166.1</td>
<td>102</td>
<td>192</td>
<td>768</td>
</tr>
<tr>
<td>Improved Window</td>
<td>4</td>
<td>32</td>
<td>2</td>
<td>$1.2 \times 10^{-4}$</td>
<td>85.7</td>
<td>85</td>
<td>165</td>
<td>160</td>
</tr>
<tr>
<td>Improved Window</td>
<td>3</td>
<td>32</td>
<td>4</td>
<td>$1.3 \times 10^{-4}$</td>
<td>88.8</td>
<td>68</td>
<td>132</td>
<td>256</td>
</tr>
<tr>
<td>Improved Window</td>
<td>2</td>
<td>32</td>
<td>8</td>
<td>$1.2 \times 10^{-4}$</td>
<td>78.3</td>
<td>51</td>
<td>99</td>
<td>384</td>
</tr>
<tr>
<td>Improved Window</td>
<td>1</td>
<td>32</td>
<td>32</td>
<td>$1.0 \times 10^{-3}$</td>
<td>77.5</td>
<td>34</td>
<td>66</td>
<td>1024</td>
</tr>
</tbody>
</table>

The detailed PSF design and simulation parameters in the comparison of high-dynamic and non-dynamic pulse-shaped signals shown in the Figure 1(e) are listed in Table IV.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSF</td>
<td>Raised-cosine roll-off PSF</td>
</tr>
<tr>
<td>Roll-off factor</td>
<td>0.5</td>
</tr>
<tr>
<td>$f_s$</td>
<td>10 MHz</td>
</tr>
<tr>
<td>$R_0$</td>
<td>0.5 Mbps</td>
</tr>
<tr>
<td>$N$</td>
<td>2</td>
</tr>
<tr>
<td>$2I$</td>
<td>32</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>8</td>
</tr>
<tr>
<td>Doppler data rate variation model</td>
<td>Cosine function</td>
</tr>
<tr>
<td>Doppler data rate variation period</td>
<td>0.1 ms</td>
</tr>
<tr>
<td>Maximum Doppler data rate</td>
<td>0.05 Mbps</td>
</tr>
<tr>
<td>Initial Doppler data phase</td>
<td>0 rad</td>
</tr>
<tr>
<td>Window function</td>
<td>Hamming window</td>
</tr>
</tbody>
</table>