Supplemental Online Appendix

1. Generation Algorithms for the Simulated Networks

1. BA model for power-law network The BA model (Barabási and Albert 1999) with $m_0 = m = 5$ was used for the generation of the power-law networks “PL1” (network size $N = 10000$) and “PL1L” (network size $N = 50000$) in our simulation study. The generation follows the two-step procedure: (1) Growth Step: starting with a small number ($m_0$) of nodes that are all connected, at every growth step, a new node is added to connect with $m$ ($m \leq m_0$) different nodes already present in the network; (2) Preferential attachment (PA) step: the probability that a new node is connected to an existing node $i$ is proportional to the current degree $n_i$ of node $i$.

2. KR model for power-law network The KR model (Krapivsky and Redner 2001) with $m_0 = 5$, $m = 1$, and $a = 0.1$ was used for the generation of the power-law networks “PL2” (network size $N = 10000$) and “PL2L” (network size $N = 50000$) in our simulation study. The generation steps are the same as those of the BA model except that the probability that a new node is connected to an existing node with degree 1 is proportional to $a$ instead. Whereas the BA model leads to power-law networks with $\gamma \approx 3$ in the degree distribution, the setting in the KR model gives us power-law networks with $\gamma \approx 2.1708$.

3. “WS” network in Watts and Strogatz (1998) We used the algorithm in Watts and Strogatz (1999) to generate WS networks. The generation algorithm contains the following steps: Start with a ring of $N$ vertices, each connected to its $m$ nearest neighbours by
undirected edges. Next, choose a member and the connection to its nearest neighbor in a clockwise sense. With probability $p$, reconnect this connection to a member randomly chosen over the entire circle, with duplicate connections forbidden. Otherwise, leave the connection in place. Repeat this process by moving clockwise around the circle, considering each member in turn until one lap is completed. Then, consider the connections between each member and its second-nearest neighbor clockwise and repeat the previous step. Continue this process, circulating around the ring and proceeding outward to more distant neighbours after each lap, until each edge in the original lattice has been considered once. Specifically, the “WS1” (network size $N = 10000$) and “WS1L” (network size $N = 50000$) networks in our simulation study were generated with a low rewinding probability using parameters $m = 20, p = 0.1$. The “WS2” (network size $N = 10000$) and “WS2L” (network size $N = 50000$) networks were generated with a high rewinding probability using parameters $m = 20, p = 0.9$.

4. **HK model for power-cluster network** The Power-cluster networks in our simulation study were generated using the HK model (Holme and Kim, 2002). The generation algorithm of this network contains the two steps in the BA model by Barabási and Albert (1999) and one extra step, called the Triad Formation (TF) step (Holme and Kim, 2002). After the Preferential attachment (PA) step, if a connection between a new member and an existing member $i$ has been added in the PA step, then add one more connection from the new member to a randomly chosen neighbor of $i$ with probability $p_t$. Specifically, “PC1” (network size $N = 10000$) and “PC1L” (network size $N = 50000$) networks were generated using parameters $m_0 = 10, m = 2, p_t = 0.1$, and “PC2” (network size $N = 10000$) and “PC2L” (network size $N = 50000$) were generated using parameters $m_0 = 10, m = 2, p_t = 1$, where $m_0$ and $m$ are defined similarly as in power-law networks and $p_t$ denotes the triad formation probability.
2. Time Series Plots of the $\rho$ Estimates from the Proposed SEQ-MCLE Approach for the Spatial Error Model

Figure 1. The average of $\hat{\rho}(t)$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL1”.

(a) $k = 1$

(b) $k = 2$
Figure 2. The average of $\hat{\rho}^{(t)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL1L”.

(a) $k = 1$

(b) $k = 2$
Figure 3: The average of $\hat{\rho}^{(i)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL2”. (Only 60 iterations are shown, since the procedure converges really fast.)

(a) $k = 1$

(b) $k = 2$
Figure 4  The average of $\hat{\rho}^{(t)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL2L”. (Only 60 iterations are shown, since the procedure converges really fast.)

(a) $k = 1$  
(b) $k = 2$
Figure 5  The average of $\hat{\rho}^{(i)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “WS1”. (For the setting of $n$ with one seed, 30 and 50 are the same, since the maximum number of connection of a single node is less than 30.)
Figure 6  The average of $\hat{\rho}^{(i)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “WS1L”. (For the setting of $n$ with one seed, 30 and 50 are the same, since the maximum number of connection of a single node is less than 30.)
The average of $\hat{\rho}(t)$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “WS2”. (For the setting of $n$ with one seed, 30 and 50 are the same, since the maximum number of connection of a single node is less than 30.)

(a) $k = 1$

(b) $k = 2$
Figure 8 The average of $\hat{\rho}^{(i)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “WS2L”. (For the setting of $n$ with one seed, 30 and 50 are the same, since the maximum number of connection of a single node is less than 30.)

(a) $k = 1$

(b) $k = 2$
Figure 9  The average of $\hat{\rho}^{(i)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PC1”.

(a) $k = 1$

(b) $k = 2$
Figure 10: The average of $\hat{\rho}^{(i)}$ with spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PC1L”.

(a) $k = 1$

(b) $k = 2$
Figure 11  The average of $\hat{\rho}^{(t)}$ with spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PC2”. 

(a) $k = 1$

(b) $k = 2$
Figure 12  The average of $\tilde{\rho}^{(i)}$ with spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PC2L”.

(a) $k = 1$

(b) $k = 2$
Figure 13  The average of $\hat{\rho}^{(t)}$ for spatial error model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “Colla”. 

(a) $k = 1$  

(b) $k = 2$
Figure 14  The average of $\hat{\rho}^{(t)}$ for spatial error model over 30 replications when $\rho \in \{0.1,0.3,0.5,0.7,0.9\}$ in network “Flickr”.

(a) $k = 1$

(b) $k = 2$
3. Time Series Plots of the $\rho$ Estimates from the Proposed SEQ-MCLE Approach for the Spatial Lag Model

Figure 15 The average of $\hat{\rho}^{(t)}$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL1”.

(a) $k = 1$

(b) $k = 2$
Figure 16  The average of $\hat{\rho}^{(t)}$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL1L”.

(a) $k = 1$  

(b) $k = 2$
Figure 17  The average of \( \hat{\rho}(t) \) for spatial lag model over 30 replications when \( \rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\} \) in network “PL2”.

(a) \( k = 1 \)  

(b) \( k = 2 \)
Figure 18  The average of $\tilde{\rho}^{(t)}$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PL2L”.

(a) $k = 1$

(b) $k = 2$
Figure 19  The average of $\hat{\rho}^{(t)}$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “WS2”.

(a) $k = 1$  
(b) $k = 2$
Figure 20  The average of $\hat{\rho}(t)$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “WS2L”.

(a) $k = 1$

(b) $k = 2$
Figure 21: The average of $\hat{\rho}^{(t)}$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PC1”.

(a) $k = 1$

(b) $k = 2$
Figure 22  The average of $\hat{\rho}^{(t)}$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “PC1L”.

(a) $k = 1$  

(b) $k = 2$
Figure 23  The average of $\hat{\rho}(t)$ for spatial lag model over 30 replications when $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in network “Flickr”.

(a) $k = 1$

(b) $k = 2$