Sub-metropolitan Tax Competition

Online Appendix

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Abstract

This online appendix formally develops the extensions discussed in Section 5 of “Sub-metropolitan Tax Competition with Household and Capital Mobility” (smtc, hereafter). Section 1 restates the baseline first-best and second-best results of smtc. Section 2.1 introduces commuting costs and shows that for jurisdictions with low or high local labor demand, workers are still regarded as perfectly mobile and the baseline results remain. For jurisdictions with intermediate-level labor demand, however, residents work where they live; the tax on residents is then used to balance the distortions caused by the business property taxation. Section 2.2 introduces immobile landowners and shows that a single business property tax constraint leads local governments to over-provide local public goods whenever immobile residents’ marginal willingness to pay for the local public good is greater than that of mobile residents. Sections 3.1, 3.2 and 3.3 respectively discuss dissociation of business property tax bases, vertical transfers and land use restrictions, and show that such policies can correct the distortions entailed by the single business property tax constraint.

Keywords: Tax competition; Mobility; Public goods; Public inputs

JEL: H71; H72; R50; R51

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1. **Baseline results**

In this section, we recall the two main results of **SMTC** which will be referred to as the baseline results in the remainder of this online appendix. Table 1 recalls the main variable definitions of the model in **SMTC**, for a given atomistic jurisdiction \( i = 1, \ldots, n \).

### Table 1. Definition of variables and functions of the baseline model

<table>
<thead>
<tr>
<th>Functions</th>
<th>endogenous variables (in ( i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^i \equiv U(x_i, g_i, R_i) : ) utility of a resident</td>
<td>( R_i ): residents ( K_i ): capital</td>
</tr>
<tr>
<td>( F^i \equiv F(W_i, K_i, L_i, z_i) : ) production</td>
<td>( L_i ): business land ( W_i ): workers</td>
</tr>
<tr>
<td>( C^i \equiv C(g_i, z_i) : ) cost of public services</td>
<td>( \rho_i ): land rent ( x_i ): numeraire good</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>exogenous variables</th>
<th>control variables (in ( i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{L}_i ): total land</td>
<td>( \tau^R_i ): tax on residents</td>
</tr>
<tr>
<td>( r ): capital return in the metropolis</td>
<td>( \tau^K_i ): capital tax</td>
</tr>
<tr>
<td>( w ): wage rate in the metropolis</td>
<td>( \tau^L_i ): business land tax</td>
</tr>
<tr>
<td>( P ): population in the metropolis</td>
<td>( \tau^P_i ): business property tax</td>
</tr>
<tr>
<td>( K ): capital in the metropolis</td>
<td>( g_i ): local public good</td>
</tr>
<tr>
<td>( z_i ): local public input</td>
<td></td>
</tr>
</tbody>
</table>

where \( U^i \) is assumed to satisfy Assumption 1\textsubscript{SMTC}. Moreover, income from capital and land are equally distributed among all households in the economy. Then, the individual income is:

\[
y = w + \frac{rK + \sum_{i=1}^{n} \rho_i L_i}{P}
\]

#### 1.1. Separate taxes on capital and business land

In the case where local governments are allowed to choose separate tax rates on capital \( \tau^K_i \) and business land \( \tau^L_i \), the local budget constraint is

\[
\tau^R_i R_i + \tau^K_i K_i + \tau^L_i L_i = C^i
\]

and the decentralized equilibrium is characterized by Result 2\textsubscript{SMTC} that we recall for convenience:

**Results 2\textsubscript{SMTC} (Baseline with \( \tau^K_i \) and \( \tau^L_i \)).** *In equilibrium, under perfect interjurisdictional competition, local government \( i \) chooses \( \tau^R_i \), \( \tau^K_i \), \( \tau^L_i \), \( g_i \) and \( z_i \) in accordance with* 

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1. Table 1 indicates the usual first-order derivative signs. Recall that \( U^i_R < 0 \) due to congestion. The following usual second-order signs are assumed: \( F_{XX}^i < 0 \) and \( F_X^i Y > 0 \) if \( X \neq Y \); \( U^i_{xx}, U^i_{yg} > 0 \) and \( C^i_{gg}, C^i_{zz} < 0 \), where subscripts stand for derivatives (as in all this appendix). See **SMTC** for more details.

2. In all this online appendix, numbers referencing to sections, assumptions, equations and results of the main paper are indexed by **SMTC**.
the following decision rules:

\[ \tau_i^R = R_i \left( \frac{|U_i^R|}{U_i^x} \right) + \tau_i^L, \]
\[ \tau_i^K = 0, \]
\[ R_i \frac{U_i^g}{U_i^x} = C_i^g, \]
\[ F_i^z = C_i^z, \]

while \( \tau_i^L \) allows to balance the budget restriction (2), so that

\[ \tau_i^L = \frac{R_i}{C_i} \left( \frac{C_i}{C_i^R - R_i \frac{|U_i^R|}{U_i^x}} \right) > 0. \]

Notice first that the positive sign of \( \tau_i^L \) results from the assumption of scale economies in the provision of public services, \( C_i^R / R_i > R_i \frac{|U_i^R|}{U_i^x} \) (see Assumption 2). Notice also that the equilibrium level of \( \tau_i^R \) is obtained by inserting (7) into (3):

\[ \tau_i^R = \frac{L_i}{C_i} \left( \frac{C_i^g}{C_i^R - R_i \frac{|U_i^R|}{U_i^x}} \right) \]

1.2. Single business property tax

In the case where local governments are constrained to choose a single business tax rate on capital and business land, \( \tau_i^P = \tau_i^K = \tau_i^L \), the local budget constraint is

\[ \tau_i^R R_i + \tau_i^P (K_i + L_i) = C_i \]

and the decentralized equilibrium is characterized by Result 3 that we also recall for convenience:

**Results 3 (Baseline with \( \tau_i^P \)).** In equilibrium, under perfect interjurisdictional competition, local governments choose \( \tau_i^R, \tau_i^P, g_i \) and \( z_i \) in accordance with the following decision rules:

\[ \tau_i^R = R_i \left( \frac{|U_i^R|}{U_i^x} \right) + \left( 1 + \frac{K_i}{L_i} \right) \tau_i^P, \]
\[ R_i \frac{U_i^g}{U_i^x} = C_i^g, \]
\[ F_i^z - C_i^z = \varepsilon_i \left[ F_i^{Kz} - F_i^{Lz} + (F_i^{KW} - F_i^{LW}) \frac{\partial W_i}{\partial z_i} \right], \]

while \( \tau_i^P \) allows to balance the budget restriction (9), so that

\[ \tau_i^P = (1 - \kappa_i) \frac{R_i}{C_i} \left( \frac{C_i^g}{C_i^R - R_i \frac{|U_i^R|}{U_i^x}} \right) > 0, \]

where \( \kappa_i \equiv K_i / (K_i + L_i) \) denotes the capital share in the business property and \( \varepsilon_i = \frac{\tau_i^P}{K_i + L_i} \frac{F_i^{KW} - F_i^{LW}}{F_i^{KW} - F_i^{LW}} < 0 \), its elasticity with respect to \( \tau_i^P \); and \( \partial W_i / \partial z_i > 0 \) is workers’ reaction to a public input increase, given \( K_i \) and \( L_i \).}

\(^3\)Notice that we have replaced the notation \( (\partial W_i / \partial z_i)_{(K_i, L_i)} \) used in smtc by \( \partial W_i / \partial z_i \) to alleviate the exposition.
Note that the equilibrium level of $\tau_i^R$ is obtained by inserting (13) into (10), and we obtain once again (8). In order to briefly outline the main changes in local governments’ behavior due to the single business property tax constraint, let us highlight the main implications of Result 3\textsuperscript{smtc} in the following corollary:

**Results 3bis (Baseline with $\tau_i^P$).** In equilibrium, under perfect interjurisdictional competition, local governments choose $\tau_i^R$, $\tau_i^P$, $g_i$ and $z_i$ so that:

\begin{align}
\tau_i^R &= \frac{L_i}{\mathcal{L}_i} \left( \frac{C_i}{L_i} + \frac{R_i |U_i^R|}{U_i^R} \right) \quad (14) \\
0 < \tau_i^P &< \frac{R_i}{\mathcal{L}_i} \left( \frac{C_i}{R_i} - \frac{R_i |U_i^R|}{U_i^R} \right) \quad (15) \\
R_i \frac{U_i^g}{U_i^z} &= C_i^g \quad (16) \\
F_i^z < C_i^z &\quad \text{if and only if} \quad F_i^z + F_i^{KW} \frac{\partial W_i}{\partial z_i} > F_i^L + F_i^{KW} \frac{\partial W_i}{\partial z_i} \quad (17)
\end{align}

Relying on the above corollary we can summarize the central findings in smtc (Propositions 2 to 4). First, conditions (14) and (16) being strictly identical to (5) and (8), we deduce that constraining local governments to set a unique tax rate on capital and business land tax base do not alter their use of household-oriented instruments - ie. resident taxes $\tau_i^R$ and local public goods $g_i$. Second, comparing (15) to (4) and (7) shows that when local governments are constrained to tax capital and business land at the same rate, they choose to under-tax business land and over-tax capital. This setting of $\tau_i^P$ is an intermediate way allowing them to finance local public services while accounting for capital mobility. Third, condition (17) indicates that a single business property tax constraint leads local authorities to over(under)-provide the local public input, compared to the Samuelson rule (12), when the overall capital-augmenting impact of the public input is stronger (weaker) than its overall land-augmenting impact. This setting of $z_i$ allows local governments to partly offset the distortions induced by the setting of $\tau_i^P$.

2. Limited household mobility

While it is usually admitted that capital can be treated as perfectly mobile across local jurisdictions, household perfect mobility is less obvious.\textsuperscript{4} Indeed, workers incur transportation costs when commuting to work outside their jurisdictions of residence and residents also incur costs (e.g. search costs) to move to another jurisdiction.\textsuperscript{5} This section examines the effects on the baseline results in section 1 of relaxing the perfect household mobility assumption. Subsection 2.1 introduces commuting costs. Subsection 2.2 introduces a group of immobile landowners who coexist with perfectly mobile residents.

\textsuperscript{4}See Wilson (1999) for an extended survey of the capital tax competition literature. A noticeable exception is Lee (1997) in which capital is treated as imperfectly mobile.

\textsuperscript{5}Mansoorian and Myers (1993) assume that residents have attachment to their jurisdiction so that they face a psychic cost when moving to another jurisdiction.
2.1. Commuting costs

In this subsection, we relax the assumption of costless commuting across jurisdictions. Since the model does not include a spatial dimension, introducing commuting costs dependent on the distance between locations would require changes in the original framework much beyond the scope of this appendix. Thus, in the sequel, we consider a fixed cost that households incur if they choose to commute outside their jurisdiction of residence.

2.1.1. Framework

The assumptions of the baseline framework, described in section 2smtc, are changed as follows. Assume that each household of the metropolis incurs a commuting cost \( c \) if she works outside her jurisdiction of residence. The commuting cost \( c \) can be interpreted as a subscription fee to a transportation network covering the whole metropolis. Denote \( w_{\text{MAX}} \) the highest wage prevailing in the metropolis. Since jurisdictions are atomistic, \( w_{\text{MAX}} \) is exogenous from jurisdiction \( i \)'s viewpoint. Each household of the metropolis supplies its one-unit labor endowment in the jurisdiction of the metropolis offering the highest wage net of commuting costs. Hence, the total labor supply to jurisdiction \( i \) is:

\[
W_i^S = \begin{cases} 
0 & \text{if } w_i < w_{\text{MAX}} - c \\
R_i & \text{if } w_{\text{MAX}} - c \leq w_i \leq w_{\text{MAX}} \\
\infty & \text{if } w_i > w_{\text{MAX}} 
\end{cases} \tag{18a}
\]

\[
W_i^S = \begin{cases} 
0 & \text{if } w_i < w_{\text{MAX}} - c \\
R_i & \text{if } w_{\text{MAX}} - c \leq w_i \leq w_{\text{MAX}} \\
\infty & \text{if } w_i > w_{\text{MAX}} 
\end{cases} \tag{18b}
\]

The amount of labor supplied in jurisdiction \( i \) has the following stepwise shape. Condition (18a) indicates that if the wage in \( i \) is lower than the net (of commuting cost) maximum wage available elsewhere, no individual supplies her labor endowment in \( i \), since it is more advantageous to commute and work in a jurisdiction with wage \( w_{\text{MAX}} \). Notice that this condition also shows that the minimum gross wage which can prevail in the metropolis is \( w_{\text{MAX}} - c \). In the remainder of this section we assume that there exists a class of municipalities with wage reaching this lower bound, \( w_{\text{MIN}} \equiv w_{\text{MAX}} - c \). Condition (18c) indicates that if the wage in \( i \) exceeds the gross maximum wage \( w_{\text{MAX}} \), an infinitely large number of residents wish to work in \( i \). Indeed, all residents living in municipalities with wage equal to \( w_{\text{MIN}} = w_{\text{MAX}} - c \) now wish to work in \( i \). Condition (18b) indicates that if the wage in \( i \) lies between the net and the gross maximum wage,
only the $R_i$ residents of $i$ desire to work in $i$. Thus, while it is too costly for a resident of $i$ to commute to another jurisdiction, other households still prefer to work in a jurisdiction with wage $w_{\text{MAX}}$ or in their jurisdiction of residence.

Since a well-behaved production function $F^i$ is assumed, the demand for labor by firms of jurisdiction $i$, $W^D_i$, is decreasing in $w_i$. Then, in the presence of commuting costs, three types of partial equilibria, depending on the local labor demand intensity, can arise in the labor market of $i$. They are depicted in Figure 1 in which the x-axis represents the amount of workers $W_i$ in jurisdiction $i$ and the y-axis represents the wage $w_i$ prevailing in $i$.

![Figure 1. Labor market partial equilibrium in jurisdiction $i$.](image)

Equilibrium $E_i$ in Figure 1 depicts the case where firms’ demand for labor in jurisdiction $i$, $W^D_i$, is relatively low given the number of residents of $i$, $R_i$. In this equilibrium where jurisdiction $i$ exports labor, local firms’ needs for labor are so low that they can afford to provide the lowest wage accepted by residents of $i$ to work in $i$, that is $w_{\text{MAX}} - c$. On the contrary, equilibrium $\hat{E}_i$ is characterized by a relatively high local demand for labor $\hat{W}^D_i$ which induces a labor import by jurisdiction $i$. In this case, to attract workers residing in jurisdictions where the lowest wage (ie. $w_{\text{MIN}} = w_{\text{MAX}} - c$) prevails, local firms have to guarantee them the minimum gross wage they accept to commute to $i$, that is $w_{\text{MIN}} + c = w_{\text{MAX}}$. Equilibrium $\hat{E}_i$ illustrates the case of an intermediate local labor demand $\hat{W}^D_i$. In this case local firms’ labor demand is sufficiently high for them to hire all the $R_i$ residents of $i$, by accepting to pay at least the price of their outside option, $w_{\text{MAX}} - c$. However, local firms’ needs for labor are not high enough that they are willing to pay the maximum wage $w_{\text{MAX}}$ in order to attract non-resident workers.

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10In this model jurisdictions might differ in labor demand intensity due to exogenous local factors $\gamma_i$ affecting the production technology $F^i = F(K_i, W_i, L_i; \gamma_i)$. On a practical ground, these factors inducing a relatively high or low local labor demand could be geographical (e.g. seaside towns) or historical (e.g. working-class towns) for instance.

11More precisely, firms of $i$ must provide a wage equal to $w_i = w_{\text{MIN}} + c + \epsilon = w_{\text{MAX}} + \epsilon$ (with $\epsilon > 0$) to attract these residents. Indeed, if $\epsilon = 0$, they receive the same wage ($w_{\text{MIN}} = w_{\text{MAX}} - c$) whether they commute to $i$ or work in their residence location. Following (18b), they would weakly prefer to work in their home jurisdiction in this case. However, for notational convenience, we ignore $\epsilon$ since its introduction does not affect the analysis.
Depending on their need for workers, local firms are ready to pay a greater or lower wage (between $w_{\text{MAX}} - c$ and $w_{\text{MAX}}$) to the $R_i$ residents of $i$. In other words, competition of local atomistic firms for the given $R_i$ residents endogenously determines the level of $w_i \in [w_{\text{MAX}} - c; w_{\text{MAX}}]$

The understanding of the three types of partial equilibria described above - $E_i$, $\widehat{E}_i$, and $\overline{E}_i$ - allows to grasp the main distinction between this framework integrating commuting costs ($c > 0$) and the baseline model with costless commuting ($c = 0$) in \textit{smtc}. This difference lies in the existence of the $\widehat{E}$-type equilibria induced by $c > 0$. In words, the presence of commuting costs induces, in each jurisdiction $i$ a range of intermediate-level wages for which all residents of $i$ decide to work in $i$. Within this wage range, workers coincide with residents ($W_i = R_i$) and the local wage $w_i$ is endogenously determined within jurisdiction $i$.

On the contrary, whenever the local demand for labor is high (or low) enough for $E$-type (or $\overline{E}$-type) equilibria to occur, the local wage is exogenously fixed at $w_{\text{MAX}} - c$ (or $w_{\text{MAX}}$) and residents of $i$ do not necessarily work in $i$. This corresponds precisely to the baseline framework in \textit{smtc}, and Result 2\textit{smtc} and 3\textit{smtc} are unchanged:

\textbf{Result A (Commuting costs).} Consider jurisdictions with relatively low or high local labor demands. Then, the decision rules of local governments are characterized by Results 2\textit{smtc} and 3\textit{smtc}.

Interestingly, if commuting costs are low, it might well be the case that very few, if any at all, jurisdictions have their wage within the intermediate-level wage range for which residents coincide with workers. Indeed, Figure 1 shows that a $\widehat{E}$-type equilibrium is less likely to occur if $c$ is very small. Hence, the baseline model in \textit{smtc} is broadly applicable for low levels of commuting costs.

2.1.2. Separate taxes on capital and business land

In the remainder of this subsection, we focus on jurisdictions with intermediate levels of local labor demand, in order to investigate the extent to which the baseline results are affected in this case. Let us focus on a representative jurisdiction $i$ with intermediate-level labor demand. Starting from the baseline framework, we must assume now that residents coincide with workers, ie. $R_i = W_i$, and that the local wage in $i$, $w_i$, is endogenous. The other hypothesis of the model are unchanged. Then, following the same steps as for deriving expression (29\textit{smtc}), the local government’s objective can be written as:

$$F(R_i, K_i, L_i - R_i, z_i) - rK_i + [F_L(R_i, K_i, L_i - R_i, z_i) - F_W(R_i, K_i, L_i - R_i, z_i)] + \tau_i R_i - C^i$$

where $\tau_i \equiv \tau_i^R - \tau_i^L$.

It is easily shown that differentiating the above objective with respect to $t_i \in \{\tau_i^R; \tau_i^L; g_i; z_i\}$, we obtain the baseline first-order conditions (30\textit{smtc}). Only the locational responses $\partial R_i/\partial t_i$ and $\partial K_i/\partial t_i$ differ from the baseline case. They

\footnote{This case where residents work where they live is similar to what is assumed in regional tax competition models with household mobility (e.g. Wilson, 1995; Richter and Wellisch, 1996). However, these frameworks differ from the present one since they ignore residential land. This explains the differences in results with the current analysis.

\footnote{Note that $F_{\text{WR}}$ still denotes the marginal product of labor which is the first argument of $F^i \equiv F(R_i, K_i, L_i)$.}
are implicitly obtained from the following system:

\[ F_K(R_i, K_i, L_i - R_i, z_i) - \tau_i^K - r = 0, \]
\[ U[\bar{y} + F_W(R_i, K_i, L_i - R_i, z_i) - F_L(R_i, K_i, L_i - R_i, z_i) - \tau_i, g_i, R_i] - \bar{u} = 0, \]

where \( \tau_i^K \) should be replaced by \( \frac{\tau_i R_i}{K_i + L_i} \) in the case of a single business property tax constraint. Then, the decentralized equilibrium in the case where local governments are allowed to use separate taxes on capital and business land is characterized by the following result:

**Result B (Commuting costs with \( \tau_i^K \) and \( \tau_i^L \)).** Consider jurisdictions with intermediate local labor demands. In equilibrium, under perfect interjurisdictional competition, local government \( i \) chooses \( \tau_i^R, \tau_i^K, \tau_i^L, g_i \) and \( z_i \) in accordance with the baseline results (3)-(7).

**Proof.** See Section A.1.

Result B is identical to the baseline Result 2rmrc. It is not surprising since, as explained in subsection 4.2rmrc, by using both the household tax \( \tau_i^R \) and the business land tax \( \tau_i^L \), local government \( i \) can treat \( \tau_i^L \) as an undistortive tax on its fixed land endowment \( L_i \), while using \( \tau_i^R \) as an instrument to control the local population size \( R_i \). It follows that: \( \tau_i^R \) is used to internalize the marginal congestion cost and the marginal fiscal cost entailed by a new resident in the jurisdiction (condition (3)); capital is not taxed (condition (4)); and public services are provided in accordance with Samuelson rules (5) and (6).

### 2.1.3. Single business property tax

The decentralized equilibrium in the case where local governments are constrained to choose a single business tax rate on capital and business land is characterized by the following result:

**Result C (Commuting costs with \( \tau_i^P \)).** Consider jurisdictions with intermediate local labor demands. In equilibrium, under perfect interjurisdictional competition, local governments choose \( \tau_i^R, \tau_i^P, g_i \) and \( z_i \) in accordance with the following decision rules:

\[ \tau_i^R < \frac{L_i}{C_i} \left( \frac{C_i}{L_i} + R_i \frac{U_i R_i}{U_x} \right) \]
\[ 0 < \tau_i^P < \frac{R_i}{C_i} \left( \frac{C_i}{R_i} - R_i \frac{U_i R_i}{U_x} \right) \]
\[ R_i \frac{U_i}{U_x} = C_i \]
\[ F_i^z < C_i \]

**Proof.** See Section A.1.

Result C shows that, in the presence of a single business property tax constraint, governments in jurisdictions with an intermediate-level labor demand (referred to as \( \tilde{E} \)-type
jurisdictions) behave differently from governments in jurisdictions with high(low)-level labor demand (referred to as $E−$type jurisdictions), whose behavior is not affected by the presence of commuting costs (see Result A). Thus, to understand the consequences of commuting costs in the model, the remainder of this subsection assumes that commuting costs exist, i.e. $c > 0$, and compares the constrained decision rules, i.e. $\tau_i^P \equiv \tau_i^K = \tau_i^L$, made in $E−$type jurisdictions, characterized by Result C, with those made in $F−$type jurisdictions, characterized by Result 3bis.

The comparison of conditions (22) and (23) with the baseline conditions (14) and (15) shows that the existence of commuting costs does not substantially affect the way local governments choose $\tau_i^P$ and $g_i$. In other words, even for $E−$type jurisdictions, $\tau_i^P$ still has to not only account for capital mobility but also enable the local governments to satisfy their budget constraints, which leads to an over(under)-taxation of business land (capital), as condition (22) indicates. Moreover, residents’ mobility still provides local governments with the right incentives to internalize their preferences regarding the amount of local public good provided $g_i$, as condition (23) shows.

However, the existence of commuting costs alters local governments’ use of $\tau_i^R$ and $z_i$, as can be seen by comparing conditions (21) and (24) with the baseline conditions (14) and (17). Specifically, in $E−$type jurisdictions, commuting costs lead local authorities to offset the business property tax distortions by distorting both $\tau_i^R$ and $z_i$ - according to (21) and (24). However, in $F−$type jurisdictions (or absent commuting costs), local governments only distort $z_i$ - according to (17). To understand this difference, we must make the economic meaning of conditions (21) and (24) clear.

Let us begin with condition (21). Commuting costs imply that residents and workers are the same persons in $E−$type jurisdictions ($E−$type equilibria in Figure 1). In this case, the tax on residents $\tau_i^R$ is a relevant instrument to balance the over(under)-taxation of capital (business land) entailed by the business property tax. Indeed, cutting $\tau_i^R$ allows to attract new workers, which by input complementarity increases the marginal product of capital relative to that of business land, if $F_{KW} > F_{LW}$. Then, as stated in condition (21), residents are under-taxed if the capital augmenting impact of an additional worker is stronger than his business land augmenting impact. However, using $\tau_i^R$ to influence directly workers location is only possible in $E−$type jurisdictions.

In $E−$type jurisdictions ($E−$type or $F−$type equilibria in Figure 1), where residents and workers do not coincide, the amount of local workers is merely determined by firms’ labor demand, not by the number of residents. This explains why, as stated in condition (14), these local governments do not distort their resident tax setting.

Let us now turn to condition (24) which indicates that in $E−$type jurisdictions, $z_i$ is distorted to compensate the distortions caused by the business property tax setting. More precisely, local governments in $E−$type jurisdictions over-provide the local public input $z_i$ if and only if raising $z_i$ directly increases the marginal product of capital relative to...
to that of business land (that is, if $F_{Kz} > F_{Lz}$).\footnote{Symmetrically, decreasing $z_i$ increases the marginal product of capital relative to that of business land, if $F_{Kz} < F_{Lz}$, which explains the under-provision in this case.} This should be contrasted with the local public input provision rule in $\hat{E}$-type jurisdictions. Indeed, condition (17) shows that governments of $\hat{E}$-type jurisdictions, when deciding on the level of $z_i$, not only account for the direct impact of the public input on capital and business land productivity (as governments of $\hat{E}$-type jurisdictions) but they also account for the attraction exerted by the public input on workers, $\partial W_i / \partial z_i$. This difference in governments’ behavior is not surprising. As seen above, in $\hat{E}$-type jurisdictions, governments can directly affect workers’ location using $\tau_i$, while governments of $E$-type jurisdictions cannot (as if commuting costs were absent). This explains why, contrary to governments of $\hat{E}$-type jurisdictions, governments of $E$-type jurisdictions use $z_i$ to influence workers’ location in order to indirectly increase the marginal product of capital relative to that of business land.\footnote{Recall that $F_{KW}, F_{LW} > 0$.}

We conclude this section by a summary of the main conclusions that can be drawn from Results A, B and C. In the presence of commuting costs, a class of jurisdictions whose workforce coincide with the local population can appear. These jurisdictions are characterized by the fact that they have an intermediate-level labor demand. When a single business property tax constraint is imposed, these jurisdictions choose to distort both their resident tax setting and their public input provision, contrary to other jurisdictions (with low or high labor demand) which behave as if commuting costs were absent by only distorting their public input provision. The basic reason for this different behavior is that commuting cost can introduce a link between residents and workers which is exploited by local authorities to control their local workforce size using their resident taxes.

2.2. Mobile and immobile residents

Assuming that all residents are perfectly mobile as in SMTC might appear as a strong assumption. In practice, jurisdictions are often composed of land owners who are relatively little mobile and renters who are much more mobile. To account for this feature, in this subsection we introduce some heterogeneity among households by adding a group of immobile residents who own all local land in their jurisdiction of residence.\footnote{To model the coexistence of households with different degree of residential mobility, we only consider the two extreme cases of infinite and no home attachment. A more general formulation would be to consider a continuum of home attachment degrees. However, introducing a Hotelling space of preferences as in e.g. Mansoorian and Myers (1993) might be complex in a model with many small jurisdictions, hence the simplifying assumption made here.} We show that this change in the model does not alter the baseline Results 2\textsuperscript{SMTC} and 3\textsuperscript{SMTC} when mobile and immobile residents have the same marginal willingness to pay (MWP) for the local public good, which arises for additively separable utility functions for instance. When MWPs can differ among the two household groups, we show that local public goods are over-provided when a single business property tax is imposed.\footnote{This heterogeneity among households is common in the literature. See e.g. Wildasin (1983) and Richter and Wellisch (1996). Also, Wildasin (1991) distinguishes between rich immobile households and poor mobile households.}
2.2.1. Framework

The assumptions of the baseline framework, described in section 2_{SMTC}, are changed as follows. We assume that jurisdiction $i$ is inhabited by $R_i^I$ identical immobile residents and $R_i^M$ identical perfectly mobile residents. Their utility is respectively $U^{Ii} \equiv U(x_i^I, g_i, R_i^I + R_i^M)$ and $U^{Mi} \equiv U(x_i^M, g_i, R_i^I + R_i^M)$. Since all local land is assumed to be possessed by immobile inhabitants, their individual budget constraints are respectively $x_i^I + \rho_i = rk + \rho_i L_i/R_i^I + w - \tau_i^R$ and $x_i^M + \rho_i = rk + w - \tau_i^R$.\footnote{Note that the only difference between immobile and mobile residents is that the former have have land income contrary to the latter. Utility functions are assumed to be identical for mobile and immobile residents.} While $R_i^I$ is exogenous, the number of mobile residents living in $i$ $R_i^M$ is endogenously determined by the free-mobility condition:

$$U(rk + w - \rho_i - \tau_i^R, g_i, R_i^I + R_i^M) = \bar{u}$$

As in the baseline model, each (mobile or immobile) household supplies its one-unit labor endowment in the jurisdiction of the metropolis with the highest wage. Since there is no commuting cost, wages are still equated across jurisdictions and exogenous due to atomicity. The land market clearing condition is:

$$R_i^I + R_i^M + L_i = \mathcal{L}_i \tag{25}$$

Since the local government cannot affect the utility of mobile residents,\footnote{Indeed, contrary to SMTC local authorities have no scope for influencing mobile residents’ satisfaction since in the present case, mobile residents do not own land.} its objective is to maximize the utility of a representative immobile resident. This can be interpreted as indicating that local public policy is controlled by immobile residents. Then, local government $i$’s problem is:

$$\max_{(\tau_i^R, \tau_i^M, g_i, z_i)} U \left( r k + \frac{\rho_i L_i}{R_i^I} + w - \rho_i - \tau_i^R, g_i, R_i^I + R_i^M \right)$$

subject to

$$\tau_i^R(R_i^I + R_i^M) + \tau_i^K K_i + \tau_i^L L_i = C(g_i, z_i) \tag{26}$$

Following the same steps as for deriving (29_{SMTC}), the local government’s objective can be written as:

$$U^{Ii} = U \left( \bar{F}_i - wW_i - rK_i + (F_i^M + \tau_i)R_i^M - C_i \right) + rk + w, g_i, R_i^I + R_i^M$$

where $\tau_i^K$ (or $\tau_i^L$ in the case of a single business property tax) has been substituted using the local budget constraint. Then, for $i \in \{\tau_i; \tau_i^K; g_i; z_i\}$, the first-order conditions of local government $i$ $dU^{Ii}/dt_i = 0$ are:

$$\left( \tau_i + R_i^M \frac{U^{Mi}_{x_i}}{U^{Ii}_{x_i}} + R_i^I \frac{U^{Ii}_{x_i}}{U^{Ii}_{x_i}} \right) \frac{\partial R_i^M}{\partial t_i} + \tau_i^K \frac{\partial K_i}{\partial t_i} + \left( R_i^M \frac{U^{Mi}_{x_i}}{U^{Mi}_{x_i}} + R_i^I \frac{U^{Ii}_{x_i}}{U^{Ii}_{x_i}} - C_i \right) \frac{\partial g_i}{\partial t_i} + (F_i - C_i) \frac{\partial z_i}{\partial t_i} = 0$$
where the same steps as for deriving (30) have been used. And the location responses \( \partial R_i^M/\partial t_i \) and \( \partial K_i/\partial t_i \) are implicitly obtained from the following system:

\[
F_W(W_i, K_i, L_i - R_i^l - R_i^M, z_i) - w = 0, \quad (27)
\]

\[
F_K(W_i, K_i, L_i - R_i^l - R_i^M, z_i) - \tau_i^K - r = 0, \quad (28)
\]

\[
U[\tau_k + w - F_L(W_i, K_i, L_i - R_i^l - R_i^M, z_i) - \tau_i, g_i, R_i^l + R_i^M] - \bar{u} = 0, \quad (29)
\]

where \( \tau_i^K \) should be replaced by \( \frac{\tau_i(R_i^l + R_i^M) - C_i}{K_i + L_i} \) in the case of a single business property tax constraint.

2.2.2. Separate taxes on capital and business land

The decentralized equilibrium in the case where local governments are allowed to use separate taxes on capital and business land is characterized by the following result:

**Result D (Mobile/Immobile residents with \( \tau_i^K \) and \( \tau_i^L \)).** In equilibrium, under perfect interjurisdictional competition, local government \( i \) chooses \( \tau_i^R, \tau_i^K, \tau_i^L, g_i \) and \( z_i \) in accordance with the following decision rules:

\[
\tau_i^R = R_i^M \left[ \frac{U_{Mi}}{U_{Mi}^x} \right] + R_i^l \left[ \frac{U_{Ri}^l}{U_{Mi}^x} \right] + \tau_i^L, \quad (30)
\]

\[
\tau_i^K = 0, \quad (31)
\]

\[
R_i^M \left[ \frac{U_{Mi}^g}{U_{Mi}^x} \right] + R_i^l \left[ \frac{U_{Ri}^l}{U_{Mi}^x} \right] = C_i^g, \quad (32)
\]

\[
F_i^z = C_i^z, \quad (33)
\]

while satisfying the budget restriction (26), so that

\[
\tau_i^L = \frac{R_i}{L_i} \left( \frac{C_i^g}{R_i} - R_i^M \left[ \frac{U_{Mi}^g}{U_{Mi}^x} \right] - R_i^l \left[ \frac{U_{Ri}^l}{U_{Mi}^x} \right] \right). \quad (34)
\]

**Proof.** See Section A.2.

Result D is qualitatively identical to the baseline result with homogenous residents (Result 2_smtc). The only change is that local government \( i \) now accounts for the differences in the MWP for the local public good among mobile and immobile residents, as well as differences in the marginal congestion cost they incur.

2.2.3. Single business property tax

The decentralized equilibrium in the case where local governments are constrained to choose a single business tax rate on capital and business land is characterized by the following result:

**Result E (Mobile/Immobile residents with \( \tau_i^P \)).** In equilibrium, under perfect interjurisdictional competition, local governments choose \( \tau_i^R, \tau_i^P, g_i \) and \( z_i \) in accordance with
the following decision rules:

\[ \tau_i^R = R_i^M \frac{U_i^{M_i}}{U_i^{R_i}} + R_i^I \frac{U_i^{I_i}}{U_i^{I_i}} + \left( 1 + \frac{K_i}{L_i} \right) \tau_i^P, \]  

\[ R_i^M \frac{U_i^g}{U_i^x} + R_i^I \frac{U_i^{I_i}}{U_i^{I_i}} < C_i^g \quad \text{if and only if} \quad \frac{U_i^{I_i}}{U_i^{I_i}} \geq \frac{U_i^g}{U_i^x}, \]  

\[ \frac{F_i^L - C_i^g}{K_i} = \varepsilon_i \left[ F_i^{K_L} - F_i^{L_L} + (F_i^{KW} - F_i^{L_W}) \frac{\partial W_i}{\partial z_i} \right], \]  

and the budget restriction (26) is satisfied, so that

\[ \tau_i^P = (1 - \kappa_i) \frac{R_i}{L_i} \left( \frac{C_i}{R_i} - R_i^M \frac{U_i^{M_i}}{U_i^{R_i}} - R_i^I \frac{U_i^{I_i}}{U_i^{I_i}} \right). \]  

Proof. See Section A.2. \qed

Conditions (35), (37) and (38) are similar to the baseline conditions (10), (12) and (13) of Result 3\textsc{smtc}. That is, the presence of immobile landowners do not alter the way local governments use tax rates on residents \( \tau_i^R \), tax rates on business property \( \tau_i^P \) and public inputs \( z_i \). Regarding the local public good provision rule (36), two cases must be distinguished.

First, when mobile and immobile residents have the same MWP for the local public good, ie. \( U_i^{I_i}/U_i^{R_i} = U_i^{M_i}/U_i^{R_i} \), condition (36) is identical to the baseline condition (11).\footnote{For example, mobile and immobile residents will have the same MWP for public goods if \( U = \phi(x + v(g, R_i^I + R_i^M)) \), which contains the common class of additively-separable utility functions as a specific case.} In this case, immobile residents who control the local policy have incentive to internalize mobile residents' preferences for local public goods since they are similar to their own preferences. It follows that the introduction of immobile landowners in the model does not alter the results of the original model if mobile and immobile residents have the same MWP for public goods.

Second, when immobile residents have a greater MWP for the local public good than mobile residents, ie. \( U_i^{I_i}/U_i^{R_i} > U_i^{M_i}/U_i^{R_i} \), condition (36) indicates that local governments over-provide local public goods.\footnote{Notice that due to the law of diminishing marginal rate of substitution, immobile residents always have a greater or equal MWP for the local public good than mobile residents, ie. \( U_i^{I_i}/U_i^{R_i} \geq U_i^{M_i}/U_i^{R_i} \).} To get the intuition behind this result, recall that when local governments are constrained to tax capital and business land at the same rate, they have to under(over)-tax business land (capital).\footnote{See Proposition 2 in \textsc{smtc}.} In other terms, they view the amount of business land used in their jurisdiction as excessively high with respect to the amount of capital.\footnote{Note that if \( \tau_i^P = 0 \), which arises absent scale economies according to (38), local governments have no incentive to distort their public good provision, ie. \( R_i^M U_i^{M_i}/U_i^x + R_i^I U_i^{I_i}/U_i^x = C_i^g \) (see (69) in the appendix section). Indeed, local governments do not distort their instruments if they can finance local public services without taxing capital.} To offset this distortion, local governments over-provide local public goods in order to attract new mobile residents and exert an upward pressure on the land rent. This acts as an incentive for firms to decrease their use of business land relative to capital.
Finally, note that this over-provision of local public goods is not observed in the baseline model because absent immobile residents, mobile residents force local governments to internalize their preferences by migrating. This is no longer true when the local public policy is controlled by immobile residents. In this case, immobile residents can take advantage of the fact that they have a greater MWP for public goods than mobile residents to over-provide the local public good.

3. Corrective policies

Regarding policy concerns, the main conclusion of SMTC is that while a unique business property tax could be justified on political and administrative grounds, it leads sub-metropolitan governments to pursue inefficient local public policies. This inefficiency requires interventions from the central government. The purpose of this section is to address the following question: which type of interventions are appropriate for tackling this inefficiency?

A single business property tax rate on capital and land inputs forces local governments, for budgetary reasons, to set an excessively high taxation of mobile capital and an inefficiently low taxation of business land in order to account for capital mobility. Two kinds of relevant interventions can then be implemented. The first type of measures allow local governments to disentangle capital and business land taxation. They are discussed in section 3.1. The second type of measures provide local governments with additional ways to finance their public service provision other than using the business property tax. Two of them are addressed in this section: vertical transfers and land use restrictions covered respectively in sections 3.2 and 3.3.

3.1. Dissociation of capital and business land taxation

The basic problem of the single business property tax rate is that it is levied on two tax bases with different mobility degrees. Since capital is perfectly mobile and entails no cost, it should not be taxed, while land, being immobile, should contribute to funding public services. Since they have only one tax instrument for these two tax bases, local governments use it half as a capital tax and half as a business land tax, which induces inefficiency. In other words, the Tinbergen principle that there should be at least as many instruments as there are objectives is not met.

Then, a natural reform which could be implemented to tackle the inefficiency caused by the business property tax rate is to replace it with two separate tax rates. However, such a reform has also drawbacks. Indeed, dissociating tax bases can entail more administrative and political costs to levy tax revenues. Moreover, tax systems are in practice very complex, which is detrimental to consent to taxation and transparency in the use of tax revenue. Hence, increasing this complexity by dissociating tax bases, which are not identical but still close, raises political concerns.

An alternative way to dissociate capital and business land taxation without increasing the number of tax rates is to remove capital from the business property tax base.

\[\text{Condition (15)}\text{ is indeed an intermediate solution between conditions (4) and (7).}\]

\[\text{See e.g. Hettich and Winer (1988, 1999) and Wilson (1995).}\]

\[\text{Such reforms have been implemented in France (2010) and in the United States (Illinois, 1979; Ohio, 2005; Michigan, 2014). See Stafford and DeBoer (2014) for a detailed discussion of such reforms in the United States.}\]
Such a reform actually boils down to imposing the optimal zero-taxation of capital and turning the business property tax into a simple land tax. Then, local authorities will be free to use the new tax to finance public services without fearing capital outflows. However, this reform also has disadvantages in practice. First, the central government usually has to compensate tax limitation reforms with vertical transfers to maintain the level of local public services. Otherwise, such measures could be too unpopular to be implemented.\footnote{For instance, all French local jurisdictions have been exactly compensated by national grant for the removal of capital from the business property tax base in 2010.} Hence, this reform might be very costly to implement and it could require levying new national taxes which might entail economic distortions. Second, eliminating the taxation of capital and having public services only rely on taxes on residents and landowners might induce negative redistributive consequences. Third, so far we have assumed that capital entails no cost when locating in a jurisdiction, which is actually not the case in practice. For instance, the use of capital by local firms can be accompanied by pollution.\footnote{See e.g. Wellisch (1995) for a model with waste emissions by local firms.} In this regard, a nationally imposed zero-tax on capital would unable local governments to bring capital owners to internalize such negative externalities. Then, a reform which addresses the inefficiency induced by single business property taxation without completely depriving local authorities of the power to tax capital might arguably be a better solution. We discuss such measures in the next two sections.

### 3.2. Vertical transfers

As discussed in the previous section, substituting single business property taxation with another tax structure has disadvantages. Thus, it might be a better solution for a central government to keep this tax in the local public policy instrument set, while providing local governments with additional ways to finance their public service provision. This would enable local governments both to lower their business property tax, thus accounting for capital mobility, and to finance the efficient level of public services. There are several ways for a central government to help local governments to finance public services.\footnote{This section and the following one discuss such methods, but other policies could be implemented such as encouraging local debt, creating various new tax instruments or developing the local public furniture of private goods and services.} The most immediate method is to directly provide them with vertical transfers to help them fulfill their budget constraints. The objective of this section is to examine the extent to which such vertical transfers allow to solve the inefficiency problem raised by the business property taxation. We show that positive vertical transfers to jurisdictions facing scale economies in the provision of their public services is a proper solution to circumvent the business property tax rigidity problem. Then, based on a numerical simulation, we discuss the shape of the optimal grant scheme depending on the population size of jurisdictions in the metropolis.

There exist many types of vertical transfers (or grants) from the central government to sub-central governments.\footnote{See e.g. Fisher (2015) for a classification of intergovernmental grants.} In this section, we introduce general nonmatching grants in the baseline framework of \textsc{smtc} with single business property taxation. Formally, each government $i$ receives a grant $S_i$ which can be used to provide public goods or public inputs (ie. general grant). Furthermore, $S_i$ does not depend on the fiscal decisions of...
the local government (ie. nonmatching grant). Then, the local budget constraint of government $i$ becomes:
\[
\tau_i^R R_i + \tau_i^P (K_i + L_i) + S_i = C^i
\]

Suppose that to finance the grant system, the central government collects a lump-sum tax $T$ from each individual which is independent of her location. The individual income net of the national tax is:
\[
y = w + \frac{rK + \sum_{i=1}^{n} \rho_i L_i}{P} - T
\]

and the grant scheme of the central government must satisfy the following budget constraint,
\[
\mathcal{PT} = \sum_{i=1}^{n} S_i
\]

All the other assumptions of the model remain the same.

As is traditional in the literature, we assume that local governments take central government policies as parametrically given. Formally, this supposes a two stage-game where in the first stage the central government (Stackelberg leader) chooses $\{S_i, T\}_{i \in [1:n]}$, and in the second stage every atomistic government $i$ (follower) chooses $\{\tau_i^R, \tau_i^P; g_i; z_i\}$ accounting for private agents’ behavior. To solve this simple game, let us start by considering the behavior of a local government $i$. Since $S_i$ and $T$ are exogenous from the viewpoint of government $i$ (and private agents), it is straightforward to show that conditions (10)-(12) in Result 3 are unchanged, and that condition (13) becomes:
\[
\tau_i^P = (1 - \kappa_i)\frac{R_i}{L_i} \left( \frac{C_i}{R_i} - \frac{R_i |U_{R_i}| / U_{x_i}}{U_{x_i} / U_{i}^i} - S_i \right)
\]

We now turn to the first stage: the central governments choice of $S_i$ and $T$. The central government can have a variety of different objectives. However, as discussed in the beginning of this section, it is assumed that the grant system is specifically implemented to help local authorities alleviate their inefficiently high taxation of capital. From (39), we can see that the central government enables local governments to use $\tau_i^P$ as an optimal zero-capital tax by providing them with a grant $S_i$ covering the excess of the per capita cost of public services $C^i / R_i$ over the marginal congestion cost $R_i |U_{R_i}| / U_{x_i}$. Then, the following result immediately arises:

**Result F (Vertical transfers). In equilibrium, under perfect interjurisdictional competition, the central government, by choosing a corrective grant scheme $\{S_i\}_{i \in [1:n]}$ such that:**
\[
\frac{S_i}{R_i} = \frac{C^i}{R_i} - \frac{R_i |U_{R_i}| / U_{x_i}}{U_{x_i} / U_{i}^i} > 0,
\]

\[34\] See e.g. Wildasin (1988) and DePeter and Myers (1994) for a tax competition model with matching grants.

\[35\] In this framework, without loss of generality, $T$ can be for instance a land tax collected by the central government in each jurisdiction $i = 1, \ldots, n$, so that $T = \sum_{i=1}^{n} \theta_i L_i / P$, where $\theta_i$ is a tax rate decided by the central government.

\[36\] Note that conditions (41)-(44) are simply conditions (10)-(13) with $\tau_i^P = 0$. 
financed by a head tax \( T = \sum_{i=1}^{n} S_i / P \), leads local governments to choose \( \tau_i^R \), \( \tau_i^P \), \( g_i \), and \( z_i \) in accordance with the following decision rules:

\[
\begin{align*}
\tau_i^R &= R_i \left| \frac{U_i^R}{U_i^x} \right|, \\
\tau_i^P &= 0, \\
R_i \frac{U_i^2}{U_i^x} &= C_i^g, \\
F_i^x &= C_i^z.
\end{align*}
\]

Result F confirms that when they are provided with the appropriate per capita grant level, local governments behave efficiently. Indeed, conditions (41), (43) and (44) coincide exactly with the first-best conditions (3), (5) and (6). Moreover the decision rule (42) characterizing the choice of the property tax \( \tau_i^P \) meets not only the first-best requirement of no capital taxation in condition (4) but also the budget clearing condition in (7) since the budget is now clearing owing to grants. In sum, Result F shows that providing local governments with an alternative way to finance the local public services breaks the ambiguity of the role played by the business property tax which can now be used as an optimal capital tax.

Condition (40) is informative about the optimal per capita grant scheme. Per capita grant must cover the gap between the per capita cost of public services and the marginal congestion cost of new residents’ consumption of public goods. The intuition behind this result is that since the resident tax \( \tau_i^R \) is specifically used to internalize congestion costs as can be seen from (41), any additional per capita cost needs to be financed by other instruments. Hence, the role of vertical transfers is precisely to take on this burden in order to free the business property tax.

From condition (40) it appears that jurisdictions with larger per capita expenditure needs (high \( C_i^g / R_i \)) should receive higher per capita grants.\(^{37}\) The literature on vertical transfers often link expenditure needs with the size of jurisdictions. Hence, to obtain more intuition about the shape of the optimal grant scheme, we provide numerical results representing per capita grant as a function of the size (ie. total population \( P \)) of metropolitan areas composed of symmetric municipalities.\(^ {38}\) We assume the following functional forms:

\[
\begin{align*}
F(W_i, K_i, L_i, z_i) &= K^\alpha W_i^b L_i^{1-a-b} z_i^d, \\
U(x_i, g_i, R_i) &= x_i + \frac{1}{\alpha} \left( g_i / R_i^{\gamma} \right)^\alpha \\
C(g_i, z_i) &= g_i + z_i + f.
\end{align*}
\]

\(^{39}\) Figure 2 represents per capita grant as a function of the population size of the metropolis.

\(^{37}\)Reschovsky (2007) properly defines expenditure needs and explains how they should drive fiscal equalization programs.

\(^{38}\)Notice that the numerical results derived hereafter remain if we consider municipalities with different sizes.

\(^{39}\)The following parameter values have been chosen:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>15</td>
</tr>
<tr>
<td>( K )</td>
<td>( P )</td>
</tr>
<tr>
<td>( L )</td>
<td>0.1P</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.001</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.005</td>
</tr>
<tr>
<td>( a )</td>
<td>0.3</td>
</tr>
<tr>
<td>( b )</td>
<td>0.65</td>
</tr>
<tr>
<td>( d )</td>
<td>0.1</td>
</tr>
<tr>
<td>( f )</td>
<td>15000</td>
</tr>
</tbody>
</table>

See section A.3 for more details about the primitive equations that lead to Figure 2 and for additional graphs.
As Figure 2a shows, the optimal corrective per capita grant scheme is U-shaped: more per capita grant should be provided to larger and smaller metropolises, in order to prevent them raising excessively high business property taxes. The intuition behind this shape is the following. In smaller metropolises, jurisdictions have a low population and thus a poor ability to raise money to finance the public services. Then, they need to be assisted to finance the high fixed costs usually required to provide public services. At the other end of the scale, in larger metropolises, crowded jurisdictions have particularly high expenditure needs which, here again, must be supported by the central authority.

This section has shown that vertical transfers to help municipalities finance their local public services can solve the inefficiency problem raised by single business property taxation. However, some limits of this solution must be pointed out. First, implementing such a grant scheme can in practice be very costly and require important rise in national taxes. Then, it might be difficult to gain acceptance by the public opinion to enforce this measure. Second, in order to provide each municipality with the appropriate level of grant, the central government needs to have a precise knowledge of its characteristics. Such an omniscience is hardly conceivable. Instead, the central government usually designs the grant scheme according to observables such as the per capita expenditure of municipalities. Then, traditional moral hazard problem can occur potentially leading municipalities to distort their provision of public services. Hence, it is not sure that a system with vertical transfers entails less distortions than a system with a single business property taxation only.

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40 Detailed discussions of various justifications for higher grants to larger jurisdictions can be found in Slack (2007, 2010) and arguments in favor of higher grants to smaller jurisdictions are discussed in Kitchen (2007).

41 Indeed, by considering that the central government acts as a Stackelberg leader, the above developments assume that the central government has a perfect knowledge of the second stage outcome.
3.3. **Land use restrictions**

In **SMTC**, competition on local land markets among residents and firms is subject to no limitation. If individuals wish to move to a jurisdiction, they simply need to be ready to pay a high enough land rent for crowding out some local firms. This free land market assumption allows to highlight local governments’ public finance choices when they face a strong tax base sensitivity due to residents’ mobility. However, this assumption is not fully realistic since local authorities usually have various instruments to directly regulate residents’ and firms’ land use in their jurisdiction (e.g. building permit, square footage cap).\(^{42}\) It is worth noticing that land use restriction policies are not a limitless power granted to local authorities. Indeed, local land use restriction policies are bounded by national laws (e.g. limited ability to expel residents and firms) and have to account for the private decisions of economic agents (e.g. granting building permits is not much useful if local land is not attractive). They are nonetheless a key instrument in local public policy and, as such, must be regarded as a strong candidate for solving the business property tax inefficiency.

This subsection examines the extent to which a land use restriction policy allows local governments to circumvent the rigidity of the instrument set imposed by the single business property taxation. For this purpose, we introduce in the baseline framework the possibility for local authorities to directly control the share of land they respectively supply to residents and firms. For simplicity, it is assumed that local land use restriction policies are not bounded by law. However, local governments account for private agents’ responses to their public policy choices. We show that land use restriction policies enable local governments to fully deal with the inefficiency arising from the single business property taxation by providing more control over tax revenues. We also describe how the introduction of land use restriction policies changes the resident tax setting and the location pattern of residents and firms.

Formally, the only assumption which differs from the baseline framework described in section 2 of **SMTC** is that local government \(i\) can directly control the supply of residential land \(R^S_i\) and the supply of business land \(L^S_i\) in its jurisdiction. That is, government \(i\) freely chooses \(R^S_i\) and \(L^S_i\), being constrained by the total local land endowment, so that:

\[
R^S_i + L^S_i = \mathcal{L}_i. \tag{45}
\]

Since residents are perfectly mobile throughout the metropolis, utility in \(i\) is still equal to the utility level \(\bar{u}\) prevailing in the metropolis. The demand for residential land \(R^D_i\) is therefore characterized by the migration condition:

\[
U(\bar{y} - \bar{\rho}_i^R - \tau_i^R, g_i, R^D_i) = \bar{u}, \tag{46}
\]

where \(\bar{y}\) is the individual income and \(\bar{\rho}_i^R\) is the residential land rent.\(^{43}\) Condition (46) defines the local demand for residential land \(R^D_i\) as a negative function of the residential local land rent \(\bar{\rho}_i^R\).\(^{44}\) The firms’ demand for land input \(L^D_i\) is characterized by the

---

\(^{42}\)See e.g. Deakin (1989), Downs (1991) and Quigley and Rosenthal (2005) for taxonomies of land use regulation policies.

\(^{43}\)Recall that, assuming that \(U(\cdot)\) satisfies Assumption 1 in **SMTC**, \(\bar{y}\) and \(\bar{u}\) are exogenous from an atomistic jurisdiction viewpoint. See section 4.1.1 in **SMTC**.

\(^{44}\)This negative relation is interpreted as follows: when \(\bar{\rho}_i^R\) increases, jurisdiction \(i\) becomes less attractive than other jurisdictions, i.e. \(U^i < \bar{u}\). Then, residents leave \(i\) to locate in another jurisdiction, i.e. \(R^D_i\) decreases. Besides, note that outflows of residents raise utility in \(i\) since they release the population density pressure in \(i\) \((U^i_R < 0\). This leads to a new migration equilibrium, i.e. \(U^i = \bar{u}\).
well-known condition:

\[ F_L(W_i, K_i, L_i^D, z_i) = \rho^L_i + \tau^P_i \]  

which defines the local demand for business land \( L_i^D \) as a negative function of the business local land rent \( \rho^L_i \).

Before proceeding, let us make clear how land use restriction policy changes the mechanisms on the local land market, compared to the free land market in SMTC.

Figure 3 represents the land market equilibrium in jurisdiction \( i \). It depicts the residential (resp. business) demand as a decreasing function of the residential (resp. business) land rent on the right (resp. left)-oriented graph. The free land market equilibrium locates at the intersection of the two demand curves \( (R_i^D) \) and \( (L_i^D) \). In this equilibrium residents and firms pay the same land rent, since they compete for the same land plots. Figure 3 also depicts the business and residential land supplies fixed by local authorities; the vertical line \( (R_i^S, L_i^S) \). The equilibrium with land use restrictions is therefore located at the respective intersections of the demand curves \( (R_i^D) \) and \( (L_i^D) \) and the supply curve \( (R_i^S, L_i^S) \). Contrary to the free market equilibrium, the residential land rent \( \rho^R_i \) differs from the business land rent \( \rho^L_i \), which reflects the absence of competition for land.\(^{45}\)

This difference in land rents is important for local authorities. To understand this, consider a policy which directly allows \( i \) to attract a small amount of residents in the two types of equilibria. In the free market equilibrium, it requires a public policy which shifts the residential land demand curve \( (R_i^D) \) towards right. In the new equilibrium both the amount of residents \( R_i \) and the residential land rent \( \rho^R_i \) will have increased by small amounts. Hence, the new residents pay almost the same land rent as the firms they have crowded out. In the equilibrium of a market with land use restrictions, attracting a small amount of residents requires a public policy which shifts the land supply curve \( (R_i^S, L_i^S) \) towards right. In the new equilibrium the amount of residents \( R_i \) and the residential (resp. business) land rent \( \rho^R_i \) (resp. \( \rho^L_i \) ) will have increased (resp. decreased)\(^{45}\).

\(^{45}\)Note that Figure 3 represents the case where \( \rho^L_i > \rho^R_i \), which means that firms pay higher land rents than households. Empirically, this is certainly the most relevant case, but the present framework does not impose such a constraint.
by small amounts. But since there exists an important land rent gap - assumed to be positive on Figure 3 - the new residents pay a much smaller land rent than firms they have crowded out. Then, a sharp and important difference exists between the two types of equilibria. In the free market equilibrium, since \( \rho_i \equiv \rho_i^R = \rho_i^L \), the crowding-out of firms by residents induces small variations in the total land rent generated in the jurisdiction \( \rho_i(R_i + L_i) = \rho_i \mathcal{L}_i \). However, when land use restriction policies are present, variations in the total land rent \( \rho_i^R R_i + \rho_i^L L_i \) induced by the crowding-out of firms by residents can become very significant due to a potentially large spread between \( \rho_i^R \) and \( \rho_i^L \). As will be seen below, this difference explains an important difference in the resident tax setting compared to the baseline result in \textit{SMTC}.

Let us return to the description of the model. Local land markets are in equilibrium so that we can denote \( R_i \equiv R_i^D = R_i^S \) and \( L_i \equiv L_i^D = L_i^S \) the equilibrium amounts of residents and business land. The remaining assumptions are similar to the baseline framework. The firms’ demand for capital and labor are characterized by:

\[
F_W(W_i, K_i, L_i, z_i) = w \tag{48}
\]

\[
F_K(W_i, K_i, L_i, z_i) = r + \tau_i^P \tag{49}
\]

In order to achieve its objective to maximize the utility of its constituents, local government \( i \) chooses \( \tau_i^R, \tau_i^P, g_i, z_i \) and \( R_i \), and satisfies the following budget constraint:

\[
(\tau_i^R - \tau_i^P)R_i + \tau_i^P(K_i + \mathcal{L}_i) = C^i \tag{50}
\]

where \( L_i \) has been substituted in (9) using (45), since choosing the amount of residents \( R_i \), government \( i \) also determines the amount of business land \( L_i \) from (45). In what follows, we assume without loss of generality that the local government freely chooses \( \tau_i^R, \tau_i^P, g_i \) and \( z_i \), while \( R_i \) adjusts endogenously so as to satisfy (50).\footnote{The results derived hereafter do not depend on the instrument used to clear the local budget constraint (50). Treating \( R_i \) as the adjustment variable makes the interpretations more suited for comparisons with \textit{SMTC}, and then allows to better understand the effect of introducing land use restrictions in the local policy instrument set.} Moreover, since the local government can directly control neither the level of the residential and business land rents nor the location of workers and capital, it has to account for responses of all these variables to its policy decisions. Formally, this means that government \( i \) has to account for the location functions implicitly \( \rho_i^R(\tau_i^R, \tau_i^P, g_i, z_i), \rho_i^L(\tau_i^R, \tau_i^P, g_i, z_i), W_i(\tau_i^R, \tau_i^P, g_i, z_i), K_i(\tau_i^R, \tau_i^P, g_i, z_i) \) and \( R_i(\tau_i^R, \tau_i^P, g_i, z_i) \) which are implicitly defined by (46) - (50).

From Lemma 1 in \textit{SMTC}, utility-maximizer local government \( i \)'s objective is to maximize the local land rent \( \rho_i^R R_i + \rho_i^L L_i \). Then, following the same steps as for deriving (29)\textsubscript{\textit{SMTC}}, the local government’s objective can be written as:

\[
F(W_i, K_i, L_i) - wW_i - r K_i + (\rho_i^R + \tau_i^R)R_i - C^i
\]

Then, for \( t_i \in \{\tau_i; \tau_i^K; g_i; z_i\} \), the first-order conditions of local government \( i \) are:

\[
\left( \frac{\tau_i^R + R_i \frac{U_i^R}{U_i^x} - \tau_i^P + \rho_i^R - \rho_i^L}{\tau_i^P \frac{\partial K_i}{\partial t_i} + \tau_i^P \frac{\partial K_i}{\partial t_i} + \left( R_i \frac{U_i^g}{U_i^x} - C^i \right) \frac{\partial g_i}{\partial t_i} + \left( F_i - C^i \right) \frac{\partial z_i}{\partial t_i} \right) = 0
\]

where the same steps as for deriving (30)\textsubscript{\textit{SMTC}} have been used.
It is informative about the mechanisms at stake in this model with land use restriction policy to have a look at the equilibrium responses of residents, workers and capital to changes in the policy instruments. It can be shown that for \( t_i \in \{g_i; z_i\}:\)

\[
\frac{\partial R_i}{\partial t_i} > 0 \quad \text{and} \quad \left\{ \begin{array}{c}
\frac{\partial K_i}{\partial g_i}, \frac{\partial W_i}{\partial g_i} < 0 \\
\frac{\partial K_i}{\partial W_i} > 0 \quad \text{(or} < 0) \\
\frac{\partial z_i}{\partial g_i}, \frac{\partial W_i}{\partial z_i} > 0 \quad \text{(or} < 0) 
\end{array} \right. \tag{52}
\]

and for \( t_i \in \{\tau_i^R; \tau_i^P\}:
\[
\frac{\partial R_i}{\partial t_i} < 0 \quad \text{and} \quad \left\{ \begin{array}{c}
\frac{\partial K_i}{\partial \tau_i^R}, \frac{\partial W_i}{\partial \tau_i^R} > 0 \\
\frac{\partial K_i}{\partial \tau_i^P}, \frac{\partial W_i}{\partial \tau_i^P} < 0 \quad \text{(or} > 0) 
\end{array} \right. \tag{53}
\]

Thus, the possibility for local governments to control their local land use significantly alters the location response pattern, as (52) and (53) show. The most important change, compared to location responses absent land use restrictions, lies in the resident location responses. Indeed, resident location now follows public budgetary rationales rather than private interests of individuals. Indeed, the local population \( R_i \) increases when additional tax revenues are needed (e.g. increase in \( g_i \) or \( z_i \)), and it decreases following a budget constraint release (e.g. increase in \( \tau_i^R \) or \( \tau_i^P \)), which confirms that local governments employ land use regulation policy as a budgetary tool. In comparison, absent land regulation, the local population increases in jurisdictions which improve their attractiveness to residents (increase in \( g_i \) or decrease in \( \tau_i^R \)) or lowers their attractiveness to firms (increase in \( \tau_i^P \) or decrease in \( z_i \)), as stated in SMTc.

Besides, note that local authorities do not face a systematic trade-off between hosting mobile residents and hosting mobile capital and workers as is the case in SMTc. The location response signs in (52) and (53) show that this trade-off still characterizes variations in the household-oriented instruments (\( g_i \) and \( \tau_i^R \)). As in SMTc, the explanation is that new households crowd out business land, which entails outflows of capital and workers (due to input complementarity). However, this trade-off is much less likely in response to variations in the firm-oriented instruments (\( z_i \) and \( \tau_i^P \)). To understand this consider a small rise in \( z_i \). By increasing the productivity of other factors, it allows to attract more capital and workers. Meanwhile, in order to finance these additional public services, the local government increases its population cap. Thus, the direct effect of raising \( z_i \) or cutting \( \tau_i^P \) is to increase both the population \( R_i \) and the amount of mobile inputs \( K_i \) and \( W_i \). This result, which sharply contrasts with SMTc, simply recalls once again that the land use regulation policy is used as a financing instrument. Finally, notice that (52) and (53) also indicate that it may happen that an increase in \( z_i \) or a cut in \( \tau_i^P \) induce outflows of capital and workers. The reason for this unusual

\[47\] See section A.4 for the derivation of these signs.

\[48\] In comparison, the location responses absent land use restrictions are (see Lemma 2 in SMTc):

\[
\frac{\partial R_i}{\partial t_i} < 0 \quad \text{and} \quad \left\{ \begin{array}{c}
\frac{\partial K_i}{\partial g_i}, \frac{\partial W_i}{\partial g_i} > 0 \\
\frac{\partial K_i}{\partial W_i}, \frac{\partial W_i}{\partial g_i} < 0 
\end{array} \right. \quad \text{for} \quad t_i \in \{\tau_i^R; \tau_i^P; g_i\}.
\]

\[49\] A small cut in \( \tau_i^P \) has the same impact.
result is that the land use policy favoring households reduces the amount of business land which in turn entails outflows of capital and workers. As a consequence, it is theoretically possible that a firm-oriented policy, which in practice is implemented to attract more capital and labor, induces the opposite effect since it needs to be financed by a population increase. However, this result requires that land increases a lot more capital and labor productivity than public inputs do. This assumption is not reasonable in the present framework which considers rough land on which households and firms build the equipment they need.

The decentralized equilibrium in the case where local governments are allowed to fix land use restrictions but are constrained to set a single business property tax rate on capital and land inputs is characterized by the following result:

**Result G (Land use restrictions).** In equilibrium, under perfect interjurisdictional competition, local governments choose $\tau^R_i, \tau^P_i, R_j, g_i$ and $z_i$ in accordance with the following decision rules:

\[ \tau^R_i = R_i \frac{U_j^R}{U_j^z} + \rho^L_i - \rho^R_i \]  
(54)
\[ \tau^P_i = 0 \]  
(55)
\[ R_i \frac{U_j^g}{U_j^z} = C^i_g \]  
(56)
\[ F^i_z = C^i_z \]  
(57)

and the residential land use $R_i$ allows to satisfy the local budget constraint so that:

\[ R_i = \frac{C^i}{\tau^R_i} \]  
(58)

**Proof.** See Section A.4.

Result G shows that allowing local governments to directly control land use in their jurisdiction leads to efficient local public policies. As conditions (56) and (57) indicate, all local public services are efficiently provided. The inefficient provision of public inputs which occurs in the absence of local land control (condition (12)) disappears. The reason for this efficient behavior is that land use restriction policy provides local authorities with the ability to finance their public service provision through the control of their population size instead of charging an inefficiently high tax on capital. Indeed, as condition (55) reveals, the business property tax can be used as a capital tax - whose desired level is zero due to capital mobility - since it does not have to play the role of a financing instrument. Indeed, this role is played by land use regulation policy, as condition (58) indicates.

Condition (54) reveals that a noticeable change occurs in the resident tax setting as compared to the first-best resident tax setting absent land use regulation characterized by condition (3). When land use restrictions are allowed, the resident tax $\tau^R_i$ does not have to internalize the marginal fiscal cost $\tau^L_i$ induced the crowding-out of a unit of business land by a new resident, since business land is not taxed anymore (condition (55)). However, the tax on residents has to internalize the gap between the business land rent and the residential land rent $\rho^L_i - \rho^R_i$. The above discussion of Figure 3 allows to
understand this. To interpret this result, assume that $\rho_L^i - \rho_R^i > 0$, which is certainly the most relevant case from an empirical viewpoint. In this case, when a new resident enters jurisdiction $i$, she entails not only a congestion cost $R_i|U_i^R/U_i^x$ but she also induces a decrease in the total land rent of the jurisdiction since she pays a lower land rent than businesses she crowded out. Yet, recall that the objective of the benevolent local government is precisely to maximize the total land rent generated in its jurisdiction. This explains that it raises $\tau^L_i$ to above the marginal congestion cost in order to bring mobile residents to internalize this land rent loss.

Finally, let us conclude this section with a summary of its main findings. The first of these is that land use restriction policy, as an additional budgetary instrument, allows to solve the inefficiency problem caused by business property taxation. It prevents local governments from setting inefficiently high taxes on capital in order to finance public services. In practice, land use restriction policy has an important place in the range of local public policy instruments. Our analysis sheds light on their beneficial role to balance the inevitable rigidity of local tax systems. However, as discussed in the beginning of this section, national laws usually limit the ability of local governments to control local land use. While these limitations are certainly justified on social and political grounds, our analysis suggests that economic gains could be obtained from providing local governments with more autonomy regarding land use regulation.

References


**Appendix**

**A.1. Commuting costs**

**A.1.1. First-best public policy rules**

Let us prove Result B. The location system (19)-(20) implicitly defines $R(\tau, \tau^K, g, z)$ and $K(\tau, \tau^K, g, z)$. Differentiating the location system (19)-(20) with respect to $\tau$ and $\tau^K$, and using Cramer’s rule, it follows that:

$$R_\tau = -|A|^{-1}F_{KK}$$

$$R_{\tau K} = |A|^{-1}(F_{WK} - F_{LK})$$

$$K_\tau = |A|^{-1}(F_{KW} - F_{KL})$$

$$K_{\tau K} = -|A|^{-1}(F_{WW} - F_{WL} - F_{LW} + F_{LL} + \frac{U_R}{U_x})$$
where
\[ A \equiv \begin{pmatrix} F_{KW} - F_{KL} \\ F_{WW} - F_{WL} - F_{LL} + U_R U_x \\ F_{WK} - F_{LK} \end{pmatrix} \]

Since
\[ \begin{vmatrix} R_\tau & R_\tau^K \\ K_\tau & K_\tau^K \end{vmatrix} \neq 0 \]

it is straightforward to show that (3)-(6) follow.\(^{50}\)

**A.1.2. Second-best public policy rules**

In this section we prove C. Replacing \( \tau_i^R \) by \( \frac{\tau_i R_i - C_i}{K_i + L_i} \) in the location system (27)-(29), differentiating with respect to \( \{\tau, g, z\} \), and using Cramer’s rule, it follows that:

\[ R_t = |B|^{-1} \begin{vmatrix} \frac{R}{K+L} \tau_t + \frac{C}{K+L} g_t + \left( -F_{Kz} + \frac{C}{K+L} \right) z_t & \frac{F_{KK}}{K+L} + \frac{\tau}{K+L} \\ \tau_t - \frac{U_R}{U_x} g_t + (-F_{Wz} + F_{Lz}) z_t & F_{WK} - F_{LK} \end{vmatrix} \]

\[ K_t = |B|^{-1} \begin{vmatrix} F_{KW} - F_{KL} + \frac{\tau}{K+L} \\ F_{WW} - F_{WL} - F_{LL} + U_R U_x \end{vmatrix} \begin{vmatrix} \frac{R}{K+L} \tau_t + \frac{C}{K+L} g_t + \left( -F_{Kz} + \frac{C}{K+L} \right) z_t \\ \tau_t - \frac{U_R}{U_x} g_t + (-F_{Wz} + F_{Lz}) z_t \end{vmatrix} \]

where
\[ B \equiv \begin{pmatrix} F_{KW} - F_{KL} + \frac{\tau}{K+L} \\ F_{WW} - F_{WL} - F_{LL} + U_R U_x \end{pmatrix} \begin{pmatrix} \frac{F_{KK}}{K+L} + \frac{\tau}{K+L} \\ F_{WK} - F_{LK} \end{pmatrix} \]

**A.1.2.1. FOC with respect to \( \tau \)**

Inserting (59) and (60) into (30), it follows that:

\[ \left( \tau + R \frac{U_R}{U_x} \right) - \frac{R}{K+L} \begin{vmatrix} \frac{F_{KK}}{K+L} + \frac{\tau}{K+L} \\ F_{WK} - F_{LK} \end{vmatrix} + \tau \begin{vmatrix} F_{KW} - F_{KL} + \frac{\tau}{K+L} \\ F_{WW} - F_{WL} - F_{LL} + U_R U_x \end{vmatrix} \left( -\frac{R}{K+L} \right) = 0 \]

Achieving \( r_1 \leftarrow r_1 + \frac{R}{K+L} r_2 \), we obtain:

\[ \left( \tau + R \frac{U_R}{U_x} \right) \begin{vmatrix} 0 & F_{KK} + \frac{\tau}{K+L} + \frac{R}{K+L} (F_{WK} - F_{LK}) \\ 1 & F_{WK} - F_{LK} \end{vmatrix} \begin{vmatrix} \frac{F_{KK}}{K+L} + \frac{\tau}{K+L} \\ F_{WK} - F_{LK} \end{vmatrix} \]

\[ + \begin{vmatrix} F_{KW} - F_{KL} + \frac{\tau}{K+L} + \frac{R}{K+L} (F_{WK} - F_{LK}) \\ F_{WW} - F_{WL} - F_{LL} + U_R U_x \end{vmatrix} \left( \tau + R \frac{U_R}{U_x} \right) \begin{vmatrix} 0 \\ 1 \end{vmatrix} = 0 \]

\(^{50}\)Note that since \( F_{WK} - F_{LK} \) and \( |A|^{-1} \) can be either positive or negative, Lemma 2 in smtc does not hold anymore.
Note that the terms $\frac{\tau^P}{K+L}$ and $\frac{\tau+RU_R/U_x}{K+L}$ cancel each other. Finally, from the land market clearing condition and Euler’s formula, we obtain:

$$\tau^R = R\frac{|U_R|}{U_x} + \left[ 1 + \frac{F_{KW} - F_{LW} - (F_{KL} - F_{LL})}{F_{KK} - F_{LK}} \right] \tau^P$$

and using once again Euler’s formula,

$$\tau^R = R\frac{|U_R|}{U_x} + \left[ 1 + \frac{K}{L} + \frac{F_{KW} - F_{LW}}{F_{KK} - F_{LK}} \right] \tau^P$$

follows.

A.1.2.2. Optimal level of $\tau^R$ and $\tau^P$

Using the optimal decision rule (61) and the local budget constraint and the budget restriction (9), we obtain:

$$\tau^R = L\left( C + R\frac{|U_R|}{U_x} \right) + \frac{F_{KW} - F_{LW}}{F_{KK} - F_{LK}} \tau^P$$

and from Euler’s formula, we can write (63) as:

$$\tau^P = \frac{1}{1 + \frac{F_{LK} - F_{LL}}{F_{KL} - F_{KK}}} \frac{R}{L} \left( C + R\frac{|U_R|}{U_x} \right)$$

Finally, recalling that second (cross) derivatives of $F$ are negative (resp. positive), (21) and (22) directly follow from (62) and (64).

A.1.2.3. FOC with respect to $g$

We have:

$$R_g = |B|^{-1} \left[ \frac{C_g}{K+L} - \frac{U_R}{U_x} \right] F_{KK} + \frac{\tau^P}{K+L} \left[ \frac{C_g}{K+L} - \frac{U_R}{U_x} \right] F_{WW} - F_{KL} + \frac{\tau}{K+L} \left[ \frac{C_g}{K+L} - \frac{U_R}{U_x} \right]$$

Inserting these location responses into (30), it follows that:

$$\left( \tau + R\frac{U_R}{U_x} \right) \left[ \frac{C_g}{K+L} - \frac{U_R}{U_x} \right] F_{KK} + \frac{\tau^P}{K+L} \left[ \frac{C_g}{K+L} - \frac{U_R}{U_x} \right] F_{WW} - F_{KL} + \frac{\tau}{K+L} R\frac{U_R}{U_x} - \frac{1}{K+L} R\frac{U_R}{U_x} + \frac{C_g}{K+L} \left| B \right| \Gamma = 0$$

\footnote{It can easily be shown that:

$$(K + L)F_{KK} + L(F_{WW} - F_{LL}) = \mathcal{L}(F_{KK} - F_{LL})$$
$$(K + L)F_{KW} + L(F_{WW} - F_{LL}) = \mathcal{L}(F_{KW} - F_{LL})$$
$$(K + L)F_{KL} + L(F_{WW} - F_{LL}) = \mathcal{L}(F_{KK} - F_{LL})$$}
where \( \Gamma \equiv +R_{Ug}^{Uz} - C_g \). Since the terms \( \tau^p \frac{C_z}{K+L} \) and \( \frac{1}{K+L} \left( \tau + R_{Ug}^{Uz} \right) \) in the determinants cancel each other, we obtain:

\[
\Pi \tau^p \left| \begin{array}{cc}
\frac{C_g}{K+L} & F_{KK} \\
-\frac{1}{Uz} & F_{WK} - F_{LK}
\end{array} \right| + \tau^p \left| \begin{array}{cc}
F_{KW} - F_{KL} - \frac{1}{K+L} R_{Ug}^{Uz} & C_g \\
F_{WW} - F_{WL} - F_{LL} + \frac{Ug}{Uz} & -\frac{1}{Uz}
\end{array} \right| + |B| \Gamma = 0
\]

where \( \tau + R_{Ug}^{Uz} \) in front of the first determinant has been replaced by \( \Pi \tau^p \) using (21) and \( \Pi \equiv 1 + \frac{F_{KW} - F_{KL} - (F_{KK} - F_{LL})}{F_{KK} - F_{LL}}. \) Performing \( c_1 \leftarrow c_1 + \Pi c_2 \) (and using (21) again) in \( |B| \) and collecting terms, we get:

\[
\left| \begin{array}{ccc}
F_{KW} - F_{KL} - \frac{1}{K+L} R_{Ug}^{Uz} - \Pi F_{KK} & \tau^p C_g + \Gamma \left( F_{KK} + \frac{\tau^p}{K+L} \right) \\
F_{WW} - F_{WL} - F_{LL} + \frac{Ug}{Uz} & -\tau^p Ug^{Uz} + \Gamma (F_{WK} - F_{LK})
\end{array} \right| = 0
\]

Operating \( r1 \leftarrow r1 + \frac{R}{K+L} r2, \) we obtain:

\[
\left| \begin{array}{ccc}
0 & \Gamma \left( F_{KK} + \frac{R}{K+Z} (F_{WK} - F_{LK}) \right) \\
F_{WW} - F_{WL} - F_{LL} + \frac{Ug}{Uz} - \Pi (F_{WK} - F_{LK}) & \tau^p Ug^{Uz} + \Gamma (F_{WK} - F_{LK})
\end{array} \right| = 0
\]

Then \( \Gamma = 0, \) which means that (23) is satisfied.

**A.1.2.4. FOC with respect to \( z \)**

We have:

\[
R_z = |B|^{-1} \left| \begin{array}{cc}
\frac{C_g}{K+L} - F_{Kz} & F_{KK} + \frac{\tau^p}{K+L} \\
F_{Lz} - F_{Wz} & F_{WK} - F_{LK}
\end{array} \right|, \quad K_z = |B|^{-1} \left| \begin{array}{cc}
F_{KW} - F_{KL} + \frac{\tau}{K+L} & C_g \\
F_{WW} - F_{WL} - F_{LL} + \frac{Ug}{Uz} & F_{Lz} - F_{Wz}
\end{array} \right|
\]

Inserting these location responses into (30), and using exactly the same computation steps as for deriving (65), it follows that:

\[
\tau^p \left[ \left( \frac{F_z}{K+L} - F_{Kz} + \frac{R}{K+L} (F_{Lz} - F_{Wz}) \right) + \Lambda \left( F_{KK} + \frac{R}{K+L} (F_{WK} - F_{LK}) \right) \right] = 0
\]

(66)

where \( \Lambda \equiv +R_{Ug}^{Uz} - C_g. \) Yet, from Euler’s formulas, we have:

\[
F_{KK} + \frac{R}{K+L} (F_{WK} - F_{LK}) = \frac{L}{K+L} (F_{KK} - F_{LK})
\]

\[
\frac{F_z}{K+L} - F_{Kz} + \frac{R}{K+L} (F_{Lz} - F_{Wz}) = \frac{L}{K+L} (F_{Lz} - F_{Kz})
\]

which inserted in (66) leads

\[
F_z^i - C_z^i = \frac{F_{Kz}^i - F_{LK}^i}{F_{KK}^i - F_{LK}^i} \tau^p
\]

from which condition (24) directly follows.
A.2. Mobile and immobile households

A.2.1. First-best public policy rules

Differentiating the location system (27)-(29) with respect to \( t \in \{ \tau, \tau^K, g, z \} \), it follows that:

\[
\begin{pmatrix}
F_{WW} & -F_{WL} & F_{WK} \\
F_{KW} & -F_{KL} & F_{KK} \\
F_{LW} & -F_{LL} - \frac{U^M}{U_x^M} & F_{LK}
\end{pmatrix}
\begin{pmatrix}
W_t \\
R^M_t \\
K_t
\end{pmatrix}
=
\begin{pmatrix}
-F_{Wz} z_t \\
\tau^K_z - F_{Kz} z_t \\
-\tau_t + \frac{U^M}{U_x^M} g_t - F_{Lz} z_t
\end{pmatrix}
\]

(67)

which is identical to system (A.7smtc). Then, the first-best location responses are the same as in the baseline model (A.11smtc)-(A.14smtc). It follows that conditions (30)-(33) can be derived following exactly the same steps as for deriving the baseline first-best conditions (3)-(6).

A.2.2. Second-best public policy rules

Using the local government’s budget constraint to substitute for \( \tau^P \) into (28) and differentiating the location system (27)-(29) with respect to \( t \in \{ \tau, g, z \} \), it follows that:

\[
\begin{pmatrix}
F_{WW} & -F_{WL} & F_{WK} \\
F_{KW} & -F_{KL} + \frac{\tau}{K+L} & F_{KK} + \frac{\tau^P}{K+L} \\
F_{LW} & -F_{LL} - \frac{U^M}{U_x^M} & F_{LK}
\end{pmatrix}
\begin{pmatrix}
W_t \\
R^P_t \\
K_t
\end{pmatrix}
=
\begin{pmatrix}
0 \\
-\frac{R^I + R^M}{K+L} \tau_t + \frac{C_g}{K+L} g_t + \left( \frac{C_g}{K+L} - \frac{U^M}{U_x^M} \right) z_t \\
\tau_t + \frac{U^M}{U_x^M} g_t - F_{Lz} z_t
\end{pmatrix}
\]

(68)

It follows that conditions (35) and (37) can be derived following exactly the same steps as for deriving the baseline first-best conditions (10) and (12). Besides, following the same steps as for deriving (A.29smtc), we obtain:

\[
\begin{vmatrix}
F_{WW} & 0 & \Gamma F_{WK} \\
F_{KW} & \frac{R^I + R^M}{K+L} U^M_x & \Gamma F_{KK} + \frac{\tau^P}{K+L} \left( R^M U^M_x + R^I U^I_x \right) \\
F_{LW} & \frac{U^M}{U_x^M} & \Gamma F_{LK} + \tau^P \frac{U^M}{U_x^M}
\end{vmatrix}
= 0
\]

where \( \Gamma \equiv R^I U^I_x + R^M U^M_x - C_g \). Performing \( r_2 \leftarrow W r_1 + (K + L) r_2 - (R^I + R^M) r_3 \), simplifying from Euler’s expressions (A.3smtc)-(A.5smtc) and collecting terms, we obtain:

\[
\begin{vmatrix}
F_{WW} & 0 & \Gamma F_{WK} \\
L (F_{KW} - F_{LW}) & 0 & \Gamma L (F_{KK} - F_{LK}) + \tau^P R^I \left( \frac{U^I_x}{U_x^M} - \frac{U^M_x}{U_x^M} \right) \\
F_{LW} & \frac{U^R}{U_x^R} & \Gamma F_{LK} + \tau^P \frac{U^M}{U_x^M}
\end{vmatrix}
= 0
\]

52 In the following appendices, we drop index \( i \) for convenience.
53 See Appendix D in smtc.
54 See Appendix E in smtc.
It follows that:

$$R^M \frac{U^M_g}{U^M_x} + R^I \frac{U^I_g}{U^I_x} - C_g = \frac{L}{K + L} \frac{F_{WW}}{D} \tau^P R^I \left( \frac{U^I_g}{U^I_x} - \frac{U^M_g}{U^M_x} \right),$$

since it is straightforward to derive from (A.9smtc) that

$$\begin{vmatrix} F_{WW} & F_{WK} \\ F_{KW} - F_{LW} & F_{KK} - F_{LK} \end{vmatrix} = \frac{K + L}{L} \begin{vmatrix} F_{WW} & F_{WK} \\ F_{KW} & F_{KK} \end{vmatrix} = \frac{K + L}{L} D.$$

Since condition (38) and scale economies (see Assumption 2 in smtc) imply $\tau^P > 0$, and second (resp. cross) derivatives of $F$ are negative (resp. positive), condition (36) follows.

### A.3. Vertical transfers

Assume the following functional forms:

$$U(x, g, R) = x + v \left( \frac{g}{R^\gamma} \right)$$

$$C(g, z) = g + z + FC$$

Note that in the main text $v(.) = 1/\alpha(\cdot)^\alpha$, but for exposition reasons, we keep the general form of $v(.)$ in the following derivations. Differentiating (70), condition (43) can be written as

$$\frac{v'(g/R^\gamma)}{R^\gamma} = 1$$

and differentiating (70), we obtain:

$$R|U_R| = R \frac{\gamma g v'(g/R^\gamma)}{R^{\gamma+1}} = \frac{\gamma g}{R}$$

From (40) and (71):

$$S = (1 - \gamma)g + z + FC$$

and condition (44) becomes:

$$F_z(K, W, L, z) = 1$$

Finally, symmetry implies that:

$$K = \frac{K}{n} \quad R = W = \frac{P}{n} \quad L = L - \frac{P}{n}$$

Then, $S/R$, $g$ and $z$ are fully determined by (72), (73) and (74), which allows to draw Figure 2 and Figure 4 hereafter:

---

Note that using the expression of $\varepsilon$ in (A.35smtc), we obtain:

$$\frac{R^M \frac{U^M_g}{U^M_x} + R^I \frac{U^I_g}{U^I_x} - C_g}{K} = \varepsilon R^I \left( \frac{U^I_g}{U^I_x} - \frac{U^M_g}{U^M_x} \right).$$
A.4. Land use restrictions

A.4.1. First-order conditions

Differentiating equations (48)-(50) and performing $r_3 \leftarrow Wr_1 + Kr_2 + Lr_3$, we obtain:

$$A \equiv \begin{pmatrix} F_{WW} & F_{WK} & 0 & 0 & -F_{WL} \\ F_{KW} & F_{KK} & 0 & 0 & -F_{KL} \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{UR}{U_z} \\ 0 & \tau^P & 0 & 0 & \tau^R - \tau^P \end{pmatrix} \begin{pmatrix} W_t \\ K_t \\ \rho_t^L \\ \rho_t^R \\ R_t \end{pmatrix} = \begin{pmatrix} -F_{Wz}z_t \\ -F_{Kz}z_t + \tau_i^P \\ -\tau_i^P \\ -\tau_i^R + \frac{U_g}{U_z}gt \\ C_ggt + C_zz_t - R\tau_i^R - (K + L)\tau_i^P \end{pmatrix}$$

Then:

$$R_{\tau_i^R} = K + L - \frac{R}{U}R_{\tau_i^R} + |A|^{-1}F_{WW}\tau^P$$

$$K_{\tau_i^P} = \frac{K + L}{R} - K_{\tau_i^R} + |A|^{-1}F_{WW}(\tau^P - \tau^R)$$

$$R_{\tau_i^R} = |A|^{-1}R(F_{KK}F_{WW} - F_{WK}^2)$$

$$K_{\tau_i^P} = |A|^{-1}R(F_{KL}F_{WW} - F_{KW}F_{WL})$$

Inserting these responses into the first-order conditions (51) with respect to $\tau_i^R$ and $\tau_i^P$, we obtain after simplification:

$$\left(\tau^R + \frac{UR}{U_x} - \tau^P + \rho^R - \rho^L\right) (F_{KK}F_{WW} - F_{WK}^2) + \tau^P(F_{KL}F_{WW} - F_{KW}F_{WL}) = 0$$

$$\left(R\frac{UR}{U_x} + \rho^R - \rho^L\right) \tau^P = 0$$

Yet $\left(R\frac{UR}{U_x} + \rho^R - \rho^L\right) \neq 0$ and $(F_{KK}F_{WW} - F_{WK}^2) \neq 0$. Then, conditions (54)-(57) follow.

A.4.2. Equilibrium location responses

Note that we have from Euler’s formula:

$$|A| = -\tau^R(F_{WW}F_{KK} - F_{WK}^2) = \tau^R\frac{L}{K}(F_{KL}F_{WW} - F_{WL}F_{WK}) < 0$$
And from (76) and using $\tau^P$, we obtain:

\[
\begin{align*}
R_{\tau R} &= -\frac{R}{\tau R} < 0, \\
R_{\tau P} &= -\frac{K + L}{\tau R} < 0, \\
R_g &= \frac{C_g}{\tau R} > 0, \\
R_z &= \frac{C_z}{\tau R} > 0,
\end{align*}
\]

\[
\begin{align*}
W_{\tau R} &= -\frac{W}{L} R_{\tau R} > 0, \\
W_{\tau P} &= -\frac{W}{L} R_{\tau P} - \frac{F_{WK}}{F_{KK} F_{WW} - F_{WK}^2} > 0 \text{ or } < 0, \\
W_g &= -\frac{W}{L} R_g < 0, \\
W_z &= -\frac{W}{L} R_z - \frac{F_{KK} F_{Wz} - F_{Kz} F_{WK}}{F_{KK} F_{WW} - F_{WK}^2} > 0 \text{ or } < 0,
\end{align*}
\]

\[
\begin{align*}
K_{\tau R} &= -\frac{K}{L} R_{\tau R} > 0, \\
K_{\tau P} &= -\frac{K}{L} R_{\tau P} + \frac{F_{WW}}{F_{KK} F_{WW} - F_{WK}^2} > 0 \text{ or } < 0, \\
K_g &= -\frac{K}{L} R_g < 0, \\
K_z &= -\frac{K}{L} R_z - \frac{F_{Kz} F_{WW} - F_{WK} F_{Wz}}{F_{KK} F_{WW} - F_{WK}^2} > 0 \text{ or } < 0.
\end{align*}
\]

since $F_{XX} < 0$ and $F_{XY} > 0$ for $X, Y \in \{K, W, L\}$. 