Electronic supplementary material.

Derivation of formulas used in the model of diapause timing, including additional analyses

Supplementing the article:

Øystein Varpe\textsuperscript{1,2,*} and Maciej J Ejsmond\textsuperscript{1,3} (2018) Trade-offs between storage and survival affect diapause timing in capital breeders. Evolutionary Ecology.

https://doi.org/10.1007/s10682-018-9961-4

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1. Calculation of fitness for an income breeder

Lifetime expected reproductive success of an income breeder $V_I$ is given by

\[ V_I(t) = p(\delta) R_I \]  

where $p$ is the probability of surviving to reproduction, dependent on adopted diapause initiation $\delta$, and $R_I$ is the expected egg production by individuals that survived to the end of the diapause period and will die before or at the end of summer. The expected reproduction is given by the definite integral $R_I$

\[ R_I = \int_{t=0}^{T} \exp(-m_a t) \left( f(t) - C_m \right) dt \]

where $t=0$ is the start of the summer season. We calculate $R_I$ by obtaining $R(t)$ i.e. the expected rate of reproduction in time

\[ R(t) = \int \exp(-m_a t) \left( f(t) - C_m \right) dt = \int \exp(-m_a t) \left( -\frac{f_p - f_T}{T_S} t + f_p - C_m \right) dt = \]

\[ \frac{f_T}{T_S} \int t \exp(-m_a t) dt - \frac{f_p}{T_S} \int \exp(-m_a t) dt + f_p \int \exp(-m_a t) dt - C_m \int \exp(-m_a t) dt = \]

\[ \exp(-m_a t) \left[ \left( t + \frac{1}{m_a} \left( \frac{f_p - f_T}{T_S} \right) - f_p + C_m \right) \right] \]

The expected reproduction for and income breeder is then given by

\[ R_I = R(T_S) - R(0) = \frac{1}{m_a} \left[ \exp(-m_a T_S) \left( \frac{f_p - f_T}{m_a T_S} + C_m - f_T \right) - \left( \frac{f_p - f_T}{m_a T_S} + C_m - f_p \right) \right] \]

2. Derivation of the condition for optimal diapause initiation in three reproductive strategies: income breeder, pure capital breeder and mixed breeder

The fitness of the three considered strategies is maximised with respect to time of diapause initiation $\delta$. The diapause initiation $\delta$ that maximizes fitness satisfy the following conditions

\[ V_I' = [p(\delta) R_I] = 0 \]

\[ V_C' = [p(\delta) R_C(\delta)] = 0 \]

\[ V_M' = [p(\delta) (R_C(\delta) + R_I)] = 0 \]
where $R_C$ and $R_I$ correspond to the offspring production covered by reserves (capital) and concurrent food intake (income), respectively. The subscripts $I$, $C$ and $M$ in the fitness measure $V$ represent the income, capital and mixed breeder respectively.

Capital breeding is the outcome of the balance between net assimilation in summer and reserves used during diapause given by formula (2.4)

$$R_C(\delta) = S_d(\delta) + S_a(\delta) =$$

$$\frac{1}{2} f(t_j) - f(\delta) - 2C_m - T_y x C_m + x C_a \delta =$$

By substituting $f(\delta) = \frac{f_T}{T_S} \delta - \frac{f_p}{T_S} \delta + f_p$ and $f(t_j) = \frac{f_T}{T_S} t_j - \frac{f_p}{T_S} t_j + f_p$ in formula (2.4) we get

$$R_C(\delta) = -\left(f_p - f_T\right) \frac{1}{2T_S} \delta^2 + \left(f_p + C_m(x - 1)\right) \delta + \left(f_p - f_T\right) \frac{T_j^2}{2T_S} - \left(f_p - C_m\right) t_j - T_y x C_m$$

and the derivative $R_C'(\delta)$ is given by

$$R_C'(\delta) = \left[S_d(\delta) + S_a(\delta)\right] = -\left(f_p - f_T\right) \frac{1}{T_S} \delta + f_p + C_m(x - 1)$$

The probability of surviving to reproduction $p$ depends on diapause timing $\delta$ and is given by

$$p(\delta) = \exp\left[-m_a(\delta - t_j) - m_d(T_y - \delta)\right]$$

with the derivative given by

$$p'(\delta) = -(m_a - m_d) \exp\left[-m_a(\delta - t_j) - m_d(T_y - \delta)\right]$$

**I. Income breeder – optimal timing**

Reproduction covered by concurrent food consumption $R_I$ is given by

$$R_I = \int_{t=0}^{T_y} \exp\left(-m_d t\right) \left(f(t) - C_m\right) dt \quad \text{(see above for the solution of the equation).}$$

By substituting equations (2.7) and (2.9) to equation (2.1), we get the condition of optimal diapause timing in income breeders, given by

$$0 = -(m_a - m_d) \exp\left[-m_a(\delta - t_j) - m_d(T_y - \delta)\right] R_I$$

Because equation 2.10 is in the product form, we can ask for the value of $\delta$ that maximizes $V_I$.

Equation 2.10 does not have a maximum and the derivative is negative irrespective of $\delta$.

Consequently, $V_I(\delta)$ is a monotonically decreasing function. The lower $\delta$ the higher value of
\( V \), and therefore organisms will tend to minimize time spent in the active phase.

Note that the expected reproductive success of an income breeder does not depend on reserves. However, an income breeder must gather enough reserves to survive the diapause period. The expected reproduction for an income breeder only depends on reserves through the probability of surviving to reproduction. To maximize \( p(\delta) \), an individual must spend the shortest possible time needed to gather the reserves sufficient to cover maintenance costs during hibernation, so \( R_C(\delta) = 0 \) (see eq. 2.5).

**II. Pure capital breeder – optimal timing**

Let us substitute equations 2.7 and 2.5 into the formula for optimal diapause initiation in pure capital breeders (equation 2.2). For convenience, we also substitute \( a = f_p - f_T \), \( b = f_p - C_m \) and \( z = m_a - m_d \). The optimal timing \( \delta \) is then given by

\[
(2.11) 0 = az\delta^2 + \left[-zT_S(b + xC_m) - a\right]2\delta - azt_j^2 + 2bT_zzt_j + 2T_S\left[b + xC_m(T_jz + 1)\right]
\]

**III. Mixed breeder – optimal timing**

For organisms able to adopt both income and capital breeding, the optimal diapause initiation is found if we substitute equations 2.5, 2.7 and 2.9 into the formula for optimal timing \( \delta \) in mixed breeders (equation 2.3). For convenience we also substitute \( a = f_p - f_T \), \( b = f_p - C_m \) and \( z = m_a - m_d \). The optimal timing \( \delta \) is then given by

\[
(2.12) 0 = az\delta^2 + \left[-zT_S(b + xC_m) - a\right]2\delta - azt_j^2 + 2bT_zzt_j + 2T_S\left[b + xC_m(T_jz + 1)\right] - 2T_zzR_j
\]

**3. Calculation of optimal levels of reserves \( \gamma \) at diapause initiation for the three reproductive strategies and constant food level**

Let us for simplicity assume that the rate of food gain does not change over the season, i.e. \( f(t) = f_p \). The total amount of reserves gathered through the juvenile phase of life \( S_a \) (see. equation 3 in the main text) can be simplified to

\[
(3.1) \delta = \frac{S_a}{f_p - C_m} + t_j
\]
By substituting equation (3.1) to formulas on the probability of surviving the diapause period $p$ (equation 6, main text) and the amount of reserves necessary to survive winter $S_d$ (equation 5, main text), we are able to express the probability $p$ and reserves needed to survive diapause $S_d$ as functions of the amount of reserves stored during the juvenile phase $S_a$ according to

$$p(S_a) = \exp\left(\frac{-(m_a - m_d)}{f_p - C_m} S_a + m_d (t_j - T_y)\right)$$

and

$$S_d = -(T_f - \delta) x C_m = -T_f x C_m + \delta x C_m = t_j x C_m - T_f x C_m + \frac{x C_m}{f_p - C_m} S_a$$

The derivatives of the function $p(S_a)$ and $S_d(S_a)$ given by equation (3.2) and (3.3) are given by

$$p(S_a)' = -\frac{m_a - m_d}{f_p - C_m} \exp\left(\frac{-(m_a - m_d)}{f_p - C_m} S_a + m_d (t_j - T_y)\right)$$

and

$$S_d(S_a)' = \frac{x C_m}{f_p - C_m}$$

The fitness of the three considered strategies (income breeder, pure capital breeder and mixed breeder) is maximised with respect to the diapause initiation strategy defined by the level of reserves $\gamma$. The model initiates diapause when the reserves given by $S_a$ reach $\gamma$. To find the $\gamma$ that maximizes fitness, we solved the conditions in which the derivative of the lifetime expected reproductive success was set to zero, given by

$$V_I(\gamma) = p(\gamma) R_I$$

$$V_C(\gamma) = p(\gamma)[\gamma + S_d(\gamma)]$$

$$V_M(\gamma) = p(\gamma)[\gamma + S_d(\gamma) + R_I]$$

where $R_I$ matches the offspring production covered by concurrent food intake (income). The subscripts $I$, $C$ and $M$ in the fitness measure $V$ mean income, capital and mixed breeder, respectively.

I. Income breeder - optimal level of reserves at diapause initiation

The fitness of an income breeder depends on diapause timing through the probability of surviving to reproduction $p$. To maximize $p(\gamma)$, an individual must spend the shortest possible
time in the feeding habitat to gather the reserves sufficient to cover maintenance costs during diapause. By substituting formula (3.3) and $b = f_p - C_m$ into the equation $S_d + \gamma = 0$ we get

\[ (3.9) \ \gamma = b \frac{x C_m (T_j - t_j)}{b + x C_m} \]

II. Pure capital breeder - optimal level of reserves at diapause initiation

In the case of a pure capital breeder, we substitute equations (3.2) and (3.3) into equation (3.7) to obtain the formula for the level of reserves that maximizes fitness. For convenience, we also substitute $z = m_a - m_d$ and $b = f_p - C_m$. This gives

\[ (3.10) \ \gamma = b \left( \frac{x C_m (T_j - t_j)}{b + x C_m} + \frac{1}{z} \right) \]

III. Mixed breeder – optimal level of reserves at diapause initiation

In case of a mixed breeder we substitute equations (3.2) and (3.3) into equation (3.8) to obtain the formula for the reserves that maximizes fitness. For convenience, we also substitute $z = m_a - m_d$ and $b = f_p - C_m$. This gives

\[ (3.11) \ \gamma = b \left( \frac{x C_m (T_j - t_j) - R_j}{b + x C_m} + \frac{1}{z} \right) \]

Note that in contrast to capital and mixed breeders, the optimal amount of reserves $\gamma$ at diapause initiation in income breeders is independent of the mortality rates $m_a$ and $m_d$.

4. Optimal diapause strategies under stage-dependent mortality rate

Motivated by the sometimes different life styles and therefore mortality risk of juvenile and adult holometabolous insects, we analyze how the optimal diapause timing is influenced when juvenile (pre-diapause) and adult (post-diapause) mortality in the active phase are allowed to differ (Fig. S1). We denote juvenile mortality $m_{aj}$ and adult mortality $m_{aa}$. Note, that in the main text we present results for $m_{aj} = m_{aa}$. 
**Fig. S1** Optimal diapause timing δ and level of reserves γ when entering diapause under stage dependent mortality i.e. juvenile mortality \( m_{\text{juv}} \) differs from adult mortality \( m_{\text{ad}} \). Income breeder (blue surface), pure capital breeder (transparent red surface) and mixed breeder (gray surface) strategies are considered under seasonally decreasing (A-C) and constant (D-F) food availability. We assumed high metabolic costs \( C_m = 0.1 \) and three different scenarios of stage-dependent mortality (explained at the panel rows). (A-C) The reserves γ do not increase proportionally with day of diapause initiation δ (note the nonlinear scale of the reserves axis) because the rate of food intake declines as the summer season progresses. (A, B, D) For legibility, the gray surface indicating δ and γ for the mixed breeder was removed when identical with the income breeder strategy.
Differences in mortality rate between juveniles versus adults do not affect diapause timing in pure capital breeders and pure income breeders as these organisms are unable to optimize the degree to which the juvenile (through storage) versus adult stage contribute to egg production (see eq. 7 and 9 in the main text). Mixed breeders may however respond, and we find that adult mortality lower than juvenile mortality moves a mixed breeder strategy towards income breeding and then also with earlier diapause (Fig. S1 A, B). On the other hand, higher mortality for adults caused mixed breeders to diapause later, store more for reproduction, and thereby behave more as capital breeders (Fig. S1 C, F).

5. Optimal diapause timing strategies when offspring value is birth-time dependent

Here we consider the optimal timing of diapause when offspring contribution to fitness is birth-time dependent. The seasonally varying offspring value would affect the reproductive success of income and mixed breeders (but not the capital breeders) because adults of these two strategies produce eggs throughout the summer (see eq. 9, 10 and Fig. 1 in the main text). To model lifetime expected reproductive success with birth-time dependent offspring contribution to fitness, we introduce a function that defines the relative offspring value \( g \) according to

\[
(5.1) \quad g = \exp(-wt)
\]

where \( w \) scales the rate of decrease of relative offspring contribution to fitness with time of birth \( t \). Eggs produced in the first day of the summer have value 1. The expected reproduction by an organism that survived diapause and where offspring fitness is birth-time dependent, then equals

\[
(5.2) \quad R_{lg} = \int_{t=0}^{T_s} \exp\left[-(m_a + w)t\right] \left(f(t) - C_m\right) dt
\]

Figure S2 displays the optimal diapause timing for two different kinds of seasonally declining offspring value (see eq. 5.1 and 5.2).
Fig. S2 Effects of introduced seasonal variation in offspring value on optimal diapause timing $\delta$ and level of reserves $\gamma$ when entering diapause. Income breeder (blue surface), pure capital breeder (transparent red surface) and mixed breeder (gray surface) strategies are modelled for seasonally decreasing (A-C) and constant (D-F) food availability. We assumed high metabolic costs $C_m=0.1$ and the following offspring value coefficient $w$ (see eq. 5.1) that defines the rate at which offspring value $g$ decreases (illustrated by the inserts): (A, D) $w=0$, (B, E) $w=0.02$, (C, F) $w=0.05$. Note that the strategy of a pure capital breeder and income breeder does not change with changing seasonality in offspring value (inserts). (A-C) The reserves $\gamma$ do not increase proportionally with day of diapause initiation $\delta$ (note the nonlinear scale of the reserves axis) because the rate of food intake declines as the summer season progresses. For legibility in panel A, the gray surface indicating $\delta$ and $\gamma$ for the mixed breeder was removed when identical with the income breeder strategy.