ONLINE RESOURCE 2: Method for Determining the Optimal Length of Protective Structures

Following federal standards, we assume that an elevation is in danger of flooding during a 100-year event if it lies below the level reached or exceeded with one percent probability by the storm waters of that event (44 CFR Part 65 2010). Any strip of coastal land located below this level would need protection. This necessitates that we compute the one-percent wave height of a 100-year flood (i.e., the wave height at the shore that could be either reached or exceeded with a 1% probability) given the local conditions and sea level. HAZUS does not provide this number directly. Instead, it combines shoreline characteristics with user-supplied 100-year stillwater elevation data to produce estimates of the significant wave height at the shoreline. Significant wave height, denoted $H_s$, is the average of the highest $1/3$ of wave heights encountered in a given region (FEMA 2009).

Longuet-Higgins (1952) demonstrates that wave heights follow approximately a Rayleigh distribution (Fig. A1). A Rayleigh distribution with parameter $\sigma > 0$ has a cumulative distribution function $F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}$. Thus, the exceedance probability is given by $1 - F(x) = e^{-\frac{x^2}{2\sigma^2}}$. We follow the method described by Goda (2010, p. 262-263) in order to transform $H_s$ obtained from HAZUS into a corresponding one-percent wave height.

Note that, for the one-percent wave height $H^*, 0.01 = e^{-\frac{(H^*)^2}{2\sigma^2}}$, i.e.,

$$H^* = \sigma \sqrt{-2\ln(0.01)}.$$  \hspace{1cm} (A.1)

Let $H_p$ denote the mean of the highest $1/p^{th}$ waves. Goda (2010) shows that if wave height follows the Rayleigh distribution,

$$\frac{H_p}{\sigma \sqrt{2}} = \sqrt{\ln(p)} + \frac{p\sqrt{\pi}}{2} \text{erfc}(\sqrt{\ln(p)}),$$  \hspace{1cm} (A.2)

Where $\text{erfc}(x)$ is the complementary error function defined as $\text{erfc}(x) = \int_x^\infty e^{-t^2} dt$.

In this notation, the significant wave height is $H_s = H_3$. Hence, plugging $p = 3$ into equation (A.2), we obtain $H_s \approx 1.42\sigma \sqrt{2}$. We can now re-write $\sigma$ as a function of $H_s$. Then, plugging the expression into equation (A.1) allows us to transform $H_s$ directly into $H^*$:

$$H^* \approx 1.52H_s.$$  \hspace{1cm} (A.3)

1 The data are obtained from flood insurance studies that are available at the Federal Emergency Management Agency (FEMA) map store at [http://store.msc.fema.gov](http://store.msc.fema.gov).
Next, we add the value for 100-year flood stillwater elevation to the one-percent wave height and obtain the total height that can be reached or exceeded by storm waters with a one-percent chance. Equipped with data on this height for every region at each point in time, we turn to the digital elevation maps. For each area lying below the one percent elevation level, we determine the optimal location and length of hard structures that would prevent surge waters from entering the area. Thus, our analysis provides us with the total length of structures needed to protect a given census block against a 100-year flood event.

References

Fig. A1 Rayleigh distribution of wave heights