Online Resource 1 – Computation of empirical membership functions

A simple way to deduce empirically the MFs from expert intensities is to count the occurrences $N_{ij}$ of each $j$-th effects at localities where the expert estimated intensity degree $i$ and to divide them by the sum of the counts of that effect over all of the intensities

$$\mu_{-1,j}(i) = \frac{N_{ij}}{\sum_{i} N_{ij}} \quad [1.1]$$

For example, if the total number of localities where a given effect is observed is 80 and in 20 of them the intensity estimated by the expert is VII, in 40 is VIII and in 20 is IX, the MF would result

$$\mu_{-1,\text{exp}}(i) = [0,0,0,0,0,0.25,0.5,0.25,0,0] \quad [1.2]$$

If intensities, which are uncertain between two adjacent degrees, are also estimated by the expert, these can be treated simply by attributing a half of a count to each of the two degrees. For example, if in the previous case there were also 20 estimates of intensity VII-VIII, we further add 10 (20/2) counts to both degrees VII (30 in total) and VIII (50 in total), so that the MF would become

$$\mu_{-1,\text{exp}}(i) = [0,0,0,0,0,0.3,0.5,0.2,0,0] \quad [1.3]$$

In order to avoid that the MFs with several non-null scores have scores smaller on average than MFs with only a few non-null scores, we normalize the MF to the maximum score

$$\mu_{-1,nj}(i) = \frac{\mu_{-1,j}(i)}{\max[\mu_{-1,j}(i)]} \quad [1.4]$$

Hence, being 0.5 the maximum score, the MF of the considered example become

$$\mu_{-1,n_{\text{exp}}}(i) = [0,0,0,0,0,0.6,1,0,0,0,0,0,0] \quad [1.5]$$

This normalization scheme, which we will indicate as “type 1”, was already adopted by Vannucci et al. (1999; 2000). However, the experience of its usage indicated that it tends to privilege the intermediate intensities (i.e. from IV to VIII) with respect to the extreme ones (<IV and >VIII) because the former ones are more frequently observed then the latter ones. In fact, let us suppose reasonably that within the entire intensity dataset of our example earthquake we have in all 20 estimates of intensity IX, 75 estimates of intensity VIII and 90 of intensity VII, the considered effect is present in all of the localities with intensity IX but only in 2/3 of the localities with intensity VIII and in 1/3 of localities with intensity VII (even considering the uncertain intensities). It would be reasonable that the membership of that effect to intensity IX should be higher than those to intensities VII and VIII and not lower as resulting from the MF of eq. [1.5].
According to such considerations, we introduce a different type of MF, which we will indicate “type 2”, where the counts $N_{ij}$ of eq. [1.1] are divided by the total number of occurrences $M_i$ of each $i$-th intensity degree for all of the localities of the given earthquake. Hence eq. [1.1] becomes

$$
\mu_{-2}(i) = \frac{N_{ij}}{M_i} = \frac{1}{\sum_i N_{ij}/M_i} [1.6]
$$

In the considered example the MF would be

$$
\mu_{-2_{exmp}}(i) = [0,0,0,0,0,0,0,17,0.33,0.5,0,0,0] [1.7]
$$

and after the application of the normalization to the maximum value (eq. [1.4]) becomes

$$
\mu_{-2_{n_{exmp}}}(i) = [0,0,0,0,0,0,0,33,0.67,1.0,0,0,0] [1.8]
$$

It is obvious that if all of the intensity values have the same frequency of occurrence in the database, type 1 MFs would be equivalent to type 2 ones. It is also evident that, as type 2 MFs weight more the least frequent intensity degrees (high and low), they are particularly liable to possible errors of association of a given effect to the given intensity value. For example the same MFs [1.7] and [1.8] would be obtained even if, for that given earthquake, there was a unique locality of intensity degree IX and the considered effect was observed at that locality (maybe erroneously). To mitigate the influence of possible mistakes and also to have an intermediate option to compare with type 1 and type 2 MFs, we define a further form of MF (“type 3”) in which the counts are divided by the square root of the number of occurrences $M_i$ (rather simply their number) of the $i$-th intensity degree for all of the localities of the considered earthquake rather than by $M_i$ as in the type 2 MFs.

$$
\mu_{-3}(i) = \frac{N_{ij}/\sqrt{M_i}}{\sum_i N_{ij}/\sqrt{M_i}} [1.9]
$$

In the considered example earthquake, the non-normalized MF would be

$$
\mu_{-3_{exmp}}(i) = [0,0,0,0,0,0,0.26,0.43,0.33,0,0,0] [1.10]
$$

while the normalized one would be

$$
\mu_{-3_{n_{exmp}}}(i) = [0,0,0,0,0,0,0.55,1.0,0.77,0,0,0] [1.11]
$$

We can note how the maximum membership score is now attributed to intensity VIII, as in the type 1 MF, but now intensity IX has a higher score than intensity VII.

As noted in the main text, when the empirical MFs show a ”multi-peak” behavior as a function of intensity, with secondary maxima separated by relative minima, we proceed to a smoothing of the
scores so that make them to decrease monotonically from the maximum to the minima. In the example shown in Fig. 1 middle panel in main text, the original MF was

$$\mu_{\text{original}}(i) = [0, 0, 0, 0, 0.07, 1.0, 0.7, 0, 0.45, 0]$$  \[1.12\]

Hence, we re-compute the score of intensity IX (0) as \((0.7+0.45)/2=0.57\) so that the smoothed MF become (Fig. 1 bottom panel in main text)

$$\mu_{\text{smooth}}(i) = [0, 0, 0, 0.07, 1.0, 0.7, 0.57, 0.45, 0]$$  \[1.13\]