Title: Kriging regression of PIV data using a local error estimate

Journal: Experiments in Fluids

Authors: Jouke H. S. de Baar, Mustafa Percin, Richard P. Dwight, Bas W. van Oudheusden, and Hester Bijl

Institute: TU Delft Address: Kluyverweg 1, 2629 HS Delft, The Netherlands

Phone: +31-15-2782596
Fax: +31-15-2787077
Email: j.h.s.debaar@tudelft.nl
Website: aerodynamics.lr.tudelft.nl/~bayesiancomputing/

The key point of this paper is that regression accuracy can be improved substantially by including observation uncertainty. This particular concept is also present in frequency-domain signal filtering of PIV data using an optimal Wiener filter, see for example [1, 2]. In this appendix, we discuss the analogy between the Kriging predictor and the Wiener filter.

Under the assumption of circularity, the estimated Kriging predictor frequency response [3–5] is exactly the response of a Wiener filter [1, 2, 6]:

\[ \Phi(f) = \frac{\hat{p}(f, \theta)}{\hat{r}(f) + \hat{p}(f, \theta)}, \]

with constant ‘white noise’ \( r \), Gaussian covariance kernel \( p \), and hats indicating Fourier transforms. The functions \( r \) and \( p \) correspond to the functions that generate the covariance matrix \( P \) and error covariance matrix \( R \) in the Kriging predictor (3).

Figure 1 is a copy of the painting “Gezicht op Delft” (View of Delft) by Jan Vermeer. Obviously, the original painting is not a digital image and was created long before Wiener became interested in signal filtering. However, what we will do is create a bitmap by making an observation of a detail of the painting. Making observations usually introduces some measurement errors, and since we are interested in local measurement errors, in Figure 2 we have created an observed bitmap with some patches of local noise. This observed bitmap will serve as our observed signal.

Both Kriging and the Wiener filter have the following objective: Use the observed signal to find an optimal reconstruction of the original signal. In Figure 2 we see the true bitmap, the observed bitmap, and the reconstructed bitmaps for different reconstruction methods. The number between brackets is the RMS Error with respect to the true bitmap. The observed bitmap has a RMSE of 0.050, while the different reconstructions reduce the RMSE.

The first observation from Figure 2 is that the Wiener filter and Kriging GE give similar reconstructions. These reconstructions are slightly different because the observed bitmap is not circular. The second observation from Figure 2 is that Kriging LE behaves different from the Wiener filter and Kriging GE. The reconstruction appears to rely less on smoothing, as is especially clear in the upper region of the bitmap. Instead of (over)smoothing, Kriging LE exploits the possibility of addressing the noise locally. As an effect, the reconstruction is more accurate and the RMSE is lower.

So why do the Wiener filter and Kriging GE behave so similar? As we have seen in (1), they have approximately the same filter response. In our case, this filter response is determined by a Gaussian kernel \( p \) and white noise \( r \). There-
Fig. 3 The accuracy of the bitmap reconstructions depends on the choice of the correlation range $\theta$ and the measurement uncertainty $\varepsilon$.

Therefore, in Wiener filtering and in Kriging GE it is equally important to estimate the signal covariance $p$ and the noise $r$, which depend on the correlation range $\theta$ and the measurement uncertainty $\varepsilon$ respectively. This is illustrated in Figure 3, where an optimal combination of $\theta$ and $\varepsilon$ minimizes the reconstruction RMSE. Deviations from this optimum lead to sub-optimal reconstructions with higher RMSEs. In this figure we see that the optimal as well as the sub-optimal reconstructions behave quite similar for the Wiener filter and Kriging GE.

In Figure 3 we also see the behavior of the Kriging LE predictor. As it only addresses the noise locally, the optimal noise level $\varepsilon$ is higher and is only applied to the noisy regions. Also, the optimal correlation range $\theta$ is shorter, such that Kriging LE will rely less on smoothing, as we already noted in Figure 2.

We conclude that for an optimal Wiener filter, it is important to match both $p$ and $r$ to the observed signal. This is closely related to the capacity of Kriging GE to optimize not only the covariance matrix $P$ but also the error covariance matrix $R$. In Kriging LE we extend this capacity by providing local error information in $R$.

References