Design and analysis of an acoustic demultiplexer exploiting negative density, negative bulk modulus and extra-ordinary transmission of membrane based acoustic metamaterial

Supplementary Material

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Derivation of Eq. (1)

The electrical equivalence of the MHR can be represented as a series capacitor-inductor circuit as shown in the below figure.

\[
C = C_V \parallel C_m
\]

- \(C_V\) - Capacitance equivalent to the volume of MHR
- \(C_m\) - Capacitance equivalent to the series combination of two membranes
- \(L\) - Inductance equivalent to the neck

The capacitance associated with the MHR is a parallel combination of capacitance equivalent to the volume of MHR \((C_V)\) and a series combination of the capacitances \((C_m)\) equivalent to the two membranes that enclose the cavity.

The capacitance equivalent to the volume of the MHR can be expressed as,

\[
C_V = \frac{ad}{B}
\]

where \(B\) is the bulk-modulus of permeated fluid \(a\) is the area of crossection of the cavity and \(d\) is the width of the cavity.

The capacitance equivalent to the membrane of the MHR can be expressed as,

\[
C_{\text{membrane}} = \frac{a^2}{\tau h r e} \quad \tau \text{ is the tension of the membrane}
\]

Then the series combination of the two membrane based capacitance can be expressed as
The effective capacitance of the MHR is

\[ C_m = \frac{a^2}{16\pi} \]

Then effective capacitance of the MHR is

\[ C = C_V + C_m \]

The inductance equivalent to the neck of the MHR can be expressed as,

\[ L = \frac{\rho L'}{a_n} \]

where \( \rho \) is the density of permeated fluid, \( L' \) is the effective length of the neck and \( a_n \) is the area of crosssection of the neck.

Then resonant frequency of the MHR is

\[ \omega_o = \frac{1}{\sqrt{LC}} = \left( \frac{16\pi B a_n}{\rho L a(Ba+16\pi d r)} \right)^{1/2} \]

**Derivation of geometrical factor, F**

The geometrical factor associated with the array of MHRs can be found out by using transmission line theory. As per the theory, the geometrical factor is the ratio between effective capacitance of the MHR and the capacitance associated with the duct-1.

The geometrical factor,

\[ F = \frac{C}{C_{duct1}} = 1 + \frac{Ba}{16\pi dr} \]

where \( C_{duct1} = \frac{ad}{B} \), the capacitance corresponding to the duct-1. \( a \) is the area of crosssection of the duct-1, \( d \) is the length of the unit cell and \( B \) is the bulk-modulus of permeated fluid.

**COMSOL Multiphysics settings for the LDM**

COMSOL Multiphysics 4.3b version has been used for the simulation. Pressure Acoustics, Frequency Domain (acpr) in Acoustic module and Membrane (mem) in Structural Mechanics module have been coupled for the analysis. Matched Boundary condition has been applied at all the four ports to avoid the reflection of acoustic waves from the boundaries. Incident pressure field is applied on port 1. Membranes were meshed with triangular meshes and remaining parts were meshed with tetrahedral meshes.