Reversible phenotypic plasticity with continuous adaptation

electronic supplementary material S: additional graphs

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Fig. S.1: Effects of the stress frequency $p$ on the maximal fitness $w$ and the corresponding optimal genotype parameter values for non-plasticity (solid lines, $m = m_0 = m_1$, $b = b_0 = b_1$), irreversible plasticity (dashed lines), instantaneous reversible plasticity (dash-dotted lines) and continuous reversible plasticity (dotted lines). The graphs for instantaneous reversible plasticity end when the stress frequency becomes too high for the model to be applicable, see the constraints (7).

Note that the ranges of the ordinates differ.

Parameter values are: $pt = 0.3$ ($t = \frac{pt}{P}$), $\phi = 1$, $\sigma_\phi = 0.3$, $v_0 = v_1 = 8$, $d_0 = d_1 = 0.1$
Fig. S.2:
Effects of the expected stress intensity $\phi$ on the maximal fitness $w$ and the corresponding optimal genotype parameter values for non-plasticity (solid lines, $m = m_0 = m_1$, $b = b_0 = b_1$), irreversible plasticity (dashed lines, almost same fitness as for non-plasticity because of high $p$), instantaneous reversible plasticity (dash-dotted lines) and continuous reversible plasticity (dotted lines).

Note that the ranges of the ordinates differ.

Parameter values are: $p = 0.7$, $pt = 0.3$ ($t = \frac{p}{pt}$), $\sigma_\phi = 0.3$, $v_0 = v_1 = 6$, $d_0 = d_1 = 0.15$.
**Fig. S.3:**
Effects of the proportion $pt$ of stress present on the maximal fitness $w$ and the corresponding optimal genotype parameter values for non-plasticity (solid lines, $m = m_0 = m_1$, $b = b_0 = b_1$), irreversible plasticity (dashed lines), instantaneous reversible plasticity (dash-dotted lines) and continuous reversible plasticity (dotted lines). The graphs of the instantaneous genotype is shown only where the model is applicable, see the constraints (7).

Note that the ranges of the ordinates differ.

Parameter values are: $p = 0.25$, $t = \frac{\phi}{p}$, $\phi = 1$, $\sigma_\phi = 0.3$, $v_0 = v_1 = 3$, $d_0 = d_1 = 0.2$
Fig. S.4:
Effects of the standard deviation \(\sigma_{\phi}\) of the stress intensities on the maximal fitness \(w\) and the corresponding optimal genotype parameter values for non-plasticity (solid lines, \(m = m_0 = m_1, b = b_0 = b_1\)), irreversible plasticity (dashed lines), instantaneous reversible plasticity (dash-dotted lines) and continuous reversible plasticity (dotted lines).

Note that the ranges of the ordinates differ.

Parameter values are: \(p = 0.25, \quad pt = 0.3 \quad (t = \frac{\phi}{p}), \quad \phi = 1, \quad v_0 = v_1 = 3, \quad d_0 = d_1 = 0.2\)
Fig. S.5:
Effects of the inverse of the transformation delay in both directions, \( \frac{1}{d} = \frac{1}{d_0} = \frac{1}{d_1} \), on the maximal fitness \( w \) and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (dash-dotted curves) in regions where \( \frac{1}{d} \) is high enough for the model to be applied, see the constraints (7), and effects of the transformation speed \( v = v_0 = v_1 \) in both directions on the maximal fitness \( w \) and corresponding optimal genotype parameter values for continuous reversible plasticity (dotted curves). The values for non-plasticity (dashed lines, \( m = m_0 = m_1, b = b_0 = b_1 \)) and irreversible plasticity (solid lines) are independent of \( \frac{1}{d} \) and \( v \) and hence the curves are horizontal.

Note that the ranges of the ordinates differ.

Parameter values are: \( p = 0.25, pt = 0.3 \ (t = \frac{pt}{p}), \phi = 1, \sigma_\phi = 0.3 \)
Fig. S.6: Effects of the stress frequency $p$ and the expected stress intensity $\bar{\phi}$ on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (left) and continuous reversible plasticity (right). For instantaneous reversible plasticity, areas where the model cannot be applied are left white, see the constraints (7).

Note that the values of the isoclines differ.

Parameter values are: $p_t = 0.3 \quad (t = \frac{pt}{p})$, $\bar{\phi} = 1$, $\sigma_{\bar{\phi}} = 0.3$, $v_0 = v_1 = 8$, $d_0 = d_1 = 0.1$
Fig. S.7:
Effects of the stress frequency $p$ and proportion $pt$ of time stress is present on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (left) and continuous reversible plasticity (right).
For instantaneous reversible plasticity, areas where the model cannot be applied are left white, see the constraints (7).

Note that the values of the isoclines differ.

Parameter values are: $t = \frac{v_t}{p}$, $\phi = 1$, $\sigma_\phi = 0.3$, $v_0 = v_1 = 8$, $d_0 = d_1 = 0.1$.
Effects of the stress frequency $p$ and the standard deviation $\sigma_\phi$ of the stress intensities on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (left) and continuous reversible plasticity (right).

For instantaneous reversible plasticity, areas where the model cannot be applied are left white, see the constraints (7).

Note that the ranges of the ordinates differ.

Parameter values are: $p_t = 0.3 \ (t = \frac{m_t}{p})$, $\phi = 1$, $d_0 = d_1 = 0.1$, $v_0 = v_1 = 8$. 

Fig. S.8:
Fig. S.9:

Left:
Effects of the expected stress frequency $p$ and the inverse $\frac{1}{d}$ of the time $d$ needed for a transformation on the maximal fitness $w$ and the corresponding optimal genotype parameter values for continuous reversible plasticity. The transformation in either direction needs the same time, $d = d_0 = d_1$. Areas where the model cannot be applied are left white, see the constraints (7).

Right:
Effects of the expected stress frequency $p$ and the transformation speed $v$ on the maximal fitness $w$ and the corresponding optimal genotype parameter values for continuous reversible plasticity. The transformations in either direction have the same speed, $v = v_0 = v_1$. At the abscissa the transformation speed is zero ($v = 0$) and hence continuous reversible plasticity resembles irreversible plasticity there.

Note that the values of the isoclines differ.

Parameter values are:
$pt = 0.3$ ($t = \frac{pt}{P}$), $\bar{\phi} = 1$, $\sigma_{\phi} = 0.3$
Fig. S.10:
Effects of the expected stress intensity $\bar{\phi}$ and the proportion $pt$ of time stress is present on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (left) and continuous reversible plasticity (right).

For instantaneous reversible plasticity, areas where the model cannot be applied are left white, see the constraints (7).

Note that the values of the isoclines differ.

Parameter values are:
\[ p = 1.3 \ (t = \frac{\bar{\phi}}{\rho}), \]
\[ \sigma_\phi = 0.3, \ v_0 = v_1 = 8, \]
\[ d_0 = d_1 = 0.1. \]
Fig. S.11: Effects of the expected stress intensity $\bar{\phi}$ and the standard deviation $\sigma_{\phi}$ of the stress intensities on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (left) and continuous reversible plasticity (right).

Note that the values of the isoclines differ.

Parameter values are: $p = 0.7$, $pt = 0.3$ ($t = \frac{pt}{p}$), $v_0 = v_1 = 6$, $d_0 = d_1 = 0.1$. 
Fig. S.12:
Left: Effects of the expected stress intensity $\phi$ and the inverse $1/d$ of the time $d$ needed for a transformation on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity. The transformation in either direction needs the same time, $d = d_0 = d_1$. Areas where the model cannot be applied are left white, see the constraints (7).

Right: Effects of the expected stress intensity $\phi$ and the transformation speed $v$ on the maximal fitness $w$ and the corresponding optimal genotype parameter values for continuous reversible plasticity. The transformations in either direction have the same speed, $v = v_0 = v_1$. At the abscissa the transformation speed is zero ($v = 0$) and hence continuous reversible plasticity reassembles the plasticity there.

Note that the values of the isoclines differ.

Parameter values are: $p = 1.4$, $pt = 0.3$ ($t = \frac{pt}{p}$), $\sigma_\phi = 0.3$. 

Fig. S.13: Effects of the proportion $pt$ of time stress is present and the standard deviation $\sigma_\phi$ of the stress intensities on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity (left) and continuous reversible plasticity (right). For instantaneous reversible plasticity, areas where the model cannot be applied are left white, see the constraints (7).

Note that the values of the isoclines differ.

Parameter values are: $p = 1.3$ ($t = \frac{pt}{p}$), $\phi = 1$, $d_0 = d_1 = 0.1$, $v_0 = v_1 = 8$
Fig. S.14:
Left: Effects of the proportion of time stress is present, $pt$, and the inverse $\frac{1}{d} = \frac{1}{d_0} = \frac{1}{d_1}$ of the transformation delay in either direction on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity. Areas where the model cannot be applied are left white; see the constraints (7).

Right: Effects of the proportion of time stress is present, $pt$, and the speed $v = v_0 = v_1$ of shifting the peak of the tolerance curve in either direction on the maximal fitness $w$ and the corresponding optimal genotype parameter values for continuous reversible plasticity.

Note that the values of the isoclines differ.

Parameter values are: $p = 1.3 \ (t = \frac{\phi}{p}), \ \phi = 1, \ \sigma_\phi = 0.2$
Fig. S.15:
Left: Effects of the standard deviation $\sigma_\phi$ of the stress intensities and the inverse $\frac{1}{\Phi}\frac{1}{d_0} = \frac{1}{\Phi}\frac{1}{d_1}$ of the transformation delay in either direction on the maximal fitness $w$ and the corresponding optimal genotype parameter values for instantaneous reversible plasticity. Areas where the model cannot be applied are left white, see the constraints (7).
Right: Effects of the standard deviation $\sigma_\phi$ of the stress intensities, and the speed $v = v_0 = v_1$ of shifting the peak of the tolerance curve in either direction on the maximal fitness $w$ and the corresponding optimal genotype parameter values for continuous reversible plasticity.

Note that the values of the isoclines differ.

Parameter values are: $p = 1.3$, $pt = 0.4$ ($t = \frac{pt}{p}$), $\phi = 1$. 
**Fig. S.16:** Upper Plot: Effect of the stress frequency $p$ on the fitness $w$ for continuous reversible plasticity with constant, not optimized, genotype parameters. Lower Plot: Length $t$ of the stress events (solid curve) and length $\frac{1}{p} - t$ of the stress-free times (dotted curve) and the durations of full transformation from the non-induced to the induced phenotype (solid horizontal) and full transformation from the induced to the non-induced phenotype (dotted horizontal), all as a function of stress frequency $p$. Parameter values are: $pt = 0.3 \ (t = \frac{pt}{p})$, $\phi = 1$, $\sigma_{\phi} = 0.3$, $v_0 = 8, v_1 = 6, m_0 = 0.1, m_1 = 0.95, b_0 = 0.6, b_1 = 0.6$

**Fig. S.17:** Upper Plot: Effect of the stress frequency $p$ on the fitness $w$ for continuous reversible plasticity with constant, not optimized, genotype parameters. Lower Plot: Length $t$ of the stress events (solid curve) and length $\frac{1}{p} - t$ of the stress-free times (dotted curve) and the durations of full transformation from the non-induced to the induced phenotype (solid horizontal) and full transformation from the induced to the non-induced phenotype (dotted horizontal), all as a function of stress frequency $p$. Parameter values are: $pt = 0.3 \ (t = \frac{pt}{p})$, $\phi = 1$, $\sigma_{\phi} = 1$, $v_0 = 8, v_1 = 6, m_0 = 0.5, m_1 = 0.5, b_0 = 0.4, b_1 = 1$
Fig. S.18: Upper Plot: Effect of the stress frequency $p$ on the fitness $w$ for continuous reversible plasticity with constant, not optimized, genotype parameters. Lower Plot: Length $t$ of the stress events (solid curve) and length $\frac{1}{p} - t$ of the stress-free times (dotted curve) and the duration of a full transformation from the non-induced to the induced phenotype and from the induced to the non-induced phenotype (overlying solid and dotted horizontals), all as a function of stress frequency $p$. Parameter values are: $pt = 0.3$ ($t = \frac{pt}{p}$), $\phi = 1$, $\sigma_\phi = 0.3$, $v_0 = 8$, $v_1 = 8$, $m_0 = 0.1$, $m_1 = 0.8$, $b_0 = 0.6$, $b_1 = 0.7$

Fig. S.19: Upper Plot: Effect of the stress frequency $p$ on the fitness $w$ for continuous reversible plasticity with constant, not optimized, genotype parameters. Lower Plot: Length $t$ of the stress events and equal length $\frac{1}{p} - t$ of the stress-free times (overlying solid and dotted curves) and the duration of a full transformation from the non-induced to the induced phenotype and from the induced to the non-induced phenotype (overlying solid and dotted horizontals), all as a function of stress frequency $p$. Parameter values are: $pt = 0.5$ ($t = \frac{pt}{p}$), $\phi = 1$, $\sigma_\phi = 0.3$, $v_0 = 8$, $v_1 = 8$, $m_0 = 0.1$, $m_1 = 0.95$, $b_0 = 0.6$, $b_1 = 0.7$
**Fig. S.20:** Effects of the single genotype parameter values on the fitness $w$ for continuous reversible plasticity when the other genotype parameter values are the optimal ones (optimized when all four parameters are free). In each graph a different parameter is drawn on the x-axis and the other genotype parameter values are displayed in the box.

Parameter values are: $p = 1.3$, $pt = 0.3$ ($t = \frac{pt}{p}$), $\phi = 1$, $\sigma_\phi = 0.3$, $v_0 = v_1 = 8$.