Supplementary Materials for “New Selection Approach for Resolution and Basis Functions in Wavelet Regression”

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1 Simulation result

1.1 Block function

For the Block function, the average MSE is summarized in supplementary material Table 1 and the choice of the primary resolution and wavelet bases is shown in supplementary material Table 2-3. The plot between the posterior probability and resolution is displayed in Figure 1 with $\sigma = 0.05$; our selected resolution is 4 when the sample size is 30 or 32 and it is 5 when the sample size is 100 or 128. However, it was 7 when the sample size is 500 or 512, while it was 9 when the sample size is 2000 or 2048. From Figure 2-4, with $\sigma = 0.1$, we have a similar result as when $\sigma = 0.05$.

1.2 Heavysine function

For the Heavysine function, the average MSE is summarized in Table 4 and the choice of the primary resolution and wavelet bases is shown in Table 5-6. Figures 5-8 show that our selected resolution is 3, while the maximum resolution increases as the sample size increases.
Table 1: Average MSE for *Block* function; “Full”=Full model with our resolution selection, all wavelet bases, and quadratic type estimation; “Reduced 1”=Reduced model with the globally selected wavelet bases and quadratic type estimation; “Reduced 2”=Reduced model with the locally selected wavelet bases and quadratic type estimation; “Full+mcmc”=Full model using MCMC samples; “Reduced1+mcmc”=Reduced 1 with mcmc; “Reduced2+mcmc”=Reduced 2 with mcmc; “Scan”=Antoniadis and Fan (2001)’s approach; “BAMS”=Vidakovic and Ruggeri (2001)’s approach; SCAN and BAMS are applicable when the sample size can express as $2^J$, while ours can be applicable for all sample size.
Table 2: The number of selected basis function using Reduced 1 approach: Given the resolution level selected for Block function, the number of basis functions selected for each resolution were counted. The bases were selected for each repetition. Selected bases were showed in all 100 repetitions.
\[ f(x) = \text{Block function} \]

<table>
<thead>
<tr>
<th>Resolution(m)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Num(n_b)</td>
<td>16</td>
<td>16</td>
<td>17</td>
<td>19</td>
<td>23</td>
<td>31</td>
<td>47</td>
<td>79</td>
<td>143</td>
<td>271</td>
<td>527</td>
</tr>
<tr>
<td>n ( \sigma )</td>
<td>30</td>
<td>0.05</td>
<td>4</td>
<td>5</td>
<td>4 \sim 6</td>
<td>4 \sim 7</td>
<td>8 \sim 11</td>
<td>0 \sim 18</td>
<td>0.1</td>
<td>5</td>
<td>4 \sim 5</td>
</tr>
</tbody>
</table>

Table 3: The number of selected basis function using Reduced 2 approach: Given the resolution level selected for Block function, the number of basis functions selected for each resolution were counted. The bases were selected for each repetition. Selected bases were showed in all 100 repetitions.
Table 4: Average MSE for *Heavysine* function; “Full”=Full model with our resolution selection, all wavelet bases and quadratic type estimation; “Reduced 1”=Reduced model with the globally selected wavelet bases and quadratic type estimation; “Reduced 2”=Reduced model with the locally selected wavelet bases and quadratic type estimation; “Full+mcmc”=Full model using MCMC samples; “Reduced1+mcmc”=Reduced 1 with mcmc; “Reduced2+mcmc”=Reduced 2 with mcmc; “Scan”=Antoniadis and Fan (2001)’s approach; “BAMS”=Vidakovic and Ruggeri (2001)’s approach; SCAN and BAMS are applicable when the sample size can express as $2^J$, while ours can be applicable for all sample size.
Table 5: The number of selected basis function using Reduced 1 approach: Given the resolution level selected for *Heavysine* function, the number of basis functions selected for each resolution were counted. The bases were selected for each repetition. Selected bases were showed in all 100 repetitions.
Table 6: The number of selected basis function using Reduced 2 approach: Given the resolution level selected for *Heavysine* function, the number of basis functions selected for each resolution were counted. The bases were selected for each repetition. Selected bases were showed in all 100 repetitions.
Figure 1: Posterior probabilities and their ratios when \( \sigma = 0.05 \) for Block function
Figure 2: Selected resolution using our approach and maximum resolution selection when $\sigma = 0.05$ for Block function
Figure 3: Posterior probabilities and their ratios when $\sigma = 0.1$ for Block function
Figure 4: Selected resolution using our approach and maximum resolution selection when $\sigma = 0.1$ for Block function.
Figure 5: Posterior probabilities and their ratios when $\sigma = 0.05$ for *Heavysine* function
Figure 6: Selected resolution using our approach and maximum resolution selection when \( \sigma = 0.05 \) for \textit{Heavysine} function
Figure 7: Posterior probabilities and their ratios when $\sigma = 0.1$ for *Heavysine* function
Figure 8: Selected resolution using our approach and maximum resolution selection when $\sigma = 0.1$ for HeavySine function