Box 1: Aerodynamic model

The combination of gravity, drag and Magnus-Robins lift forces determine the flight path of a spinning ball through the air. Forces acting on the ball can be expressed as:

\[ \sum \mathbf{F} = m \cdot g + \mathbf{F}_d + \mathbf{F}_l \]  

where \( m \) is the mass of the ball, \( g \) gravity, \( \mathbf{F}_d \) and \( \mathbf{F}_l \) drag and lift forces, respectively. \( \mathbf{F}_d \) and \( \mathbf{F}_l \) are assumed to vary with the velocity squared so that:

\[ F_d = \frac{1}{2} \cdot C_d \cdot \rho \cdot A \cdot V^2 \]  

\[ \text{And} \]

\[ F_l = \frac{1}{2} \cdot C_l \cdot \rho \cdot A \cdot V^2 \]

where \( \rho \) is the air density, \( A \) the cross-sectional area of the ball, \( C_d \) the drag coefficient, \( C_l \) the lift coefficient and \( V \) the velocity.

To determine the equations that describe the lift force on the ball, the spinning ball is considered as a two-dimensional rotating cylinder. By subsequently using the Kutta-Joukowski lift theorem for cylinders, an approximation of the magnitude of the force (\( \mathbf{F}_l \)) generated by a spinning ball can be obtained. The Kutta-Joukowski lift theorem states that the lift per unit length of a spinning cylinder is equal to the density (\( \rho \)) of the air times the strength of the rotation (\( G = 4 \cdot \pi^2 \cdot R^2 \cdot \omega \)) times the velocity (\( V \)) of the air. For a ball, the length of a "cylinder" equals twice the radius of the ball (2R). This force would act over a cross-sectional area equals to pi times the radius (R) squared. (Area=\( \pi \cdot R^2 \)). Thus, the final Magnus lift force equation can be expressed as:

\[ F_l = (\rho \cdot G \cdot V) \cdot (2 \cdot R) \cdot \frac{\pi}{4} \]
A modelling tool, based on a small time step of 10 ms computed the ball path. It was assumed that the drag and lift coefficients remained constant during the flight. The drag coefficient was taken from a series of wind-tunnel experiments while the lift coefficient was calculated using equation (4) and was also compared to other wind-tunnel experimental data.

The equations of motion are obtained by resolving the resulting acceleration (equation (1)), in the reference frame \((x, y, z)\) (see Figure 1b).

The reference frame used to calculate the ball’s flight path was such that gravity \(m \cdot g\) is directed along the \(z\) axis \((- m \cdot g \cdot z)\), the Magnus lift force is directed along the \(y''\) axis while the drag force is perpendicular to the lift and directed along the \(x''\) axis \((= x'')\). These equations were then written in terms of the relative displacements (equation (5)) and resolved using a classical finite difference scheme with a 10 ms time step.

Using the above equations (5) a large range of ball flight trajectories could be generated.

To validate the data derived from our model, direct comparisons with real free kick situations and previous models (Bray & Kerwin, 2003; de Mestre 1990) were made.

Part 2: Current arrival position computation

For each time step, the current arrival position is represented by the tangent between the trajectory and the goal line. The equation of the ball trajectory can be rewritten in the horizontal plane as a function of time \((t)\).

\[
y = f(t) = \frac{F_y}{2 \cdot m} \cdot t^2 + V_y \cdot t \quad (6)
\]

\[
x = g(t) = \frac{F_x}{2 \cdot m} \cdot t^2 + V_x \cdot t \quad (7)
\]
At a given point \((t_0, y(t_0))\), the local tangent to the ball trajectory is expressed by:

\[
y = f'(t_0) \cdot (t - t_0) + y_0
\]

with

\[
y_0 = f(t_0)
\]

Using (6), (7) and (8) to solve the system of equations in (9) the current arrival position \((y)\) at each time step \(t_o\) can be calculated:

\[
\begin{aligned}
\frac{F_x}{2 \cdot m} \cdot t^2 + V_x \cdot t - 30 &= 0 \\
y = f'(t_0) \cdot (t - t_0) + y_0
\end{aligned}
\]

(9)