

L_1

$$\hat{p}_{(j)} = \begin{cases} \frac{n_{(j)}}{\gamma}, & j=1 \\ \frac{n_{(j)}}{2\lambda d + \gamma}, & j=2 \\ \frac{n_{(j)}}{2\lambda (d+e) + \gamma}, & j=3,4 \end{cases} . \quad (14)$$

Note that $d + e = 1$ and thus, γ satisfies the following equation

$$\sum_{j=1}^4 p_{(j)} = \frac{n_{(1)}}{\gamma} + \frac{n_{(2)}}{2\lambda d + \gamma} + \frac{n_{(3)} + n_{(4)}}{2\lambda + \gamma} = 1. \quad (15)$$

We can solve for γ by taking the real roots of the cubic equation

$$\gamma^3 + A \gamma^2 + B \gamma + C = 0, \quad (16)$$

where $A = 2\lambda(1+d) - N$, $B = 2\lambda \times [-n_{(1)} - n_{(2)} - d(N - n_{(2)}) + 2\lambda d]$ and $C = -4\lambda^2 n_{(1)} d$.

L_2

$$\hat{p}_{(j)} = \begin{cases} \frac{((1+d)\lambda - \gamma) \pm \sqrt{((1+d)\lambda - \gamma)^2 + 8\lambda n_{(j)}}}{4\lambda}, & j=1 \\ \frac{(\lambda e - \gamma) \pm \sqrt{(\lambda e - \gamma)^2 + 8\lambda n_{(j)}}}{4\lambda}, & j=2 \\ \frac{-\gamma \pm \sqrt{\gamma^2 + 8\lambda n_{(j)}}}{4\lambda} & j=3,4 \end{cases} . \quad (17)$$

To solve for γ , take the sum of the positive roots for each $\hat{p}_{(j)}$.