A simple bound may be defined for the positive values of the correlation functions sidelobes in De Bruijn sequences [16]:

\[ 0 \leq \max \theta(\tau) \leq 2^n - 4 \left\lfloor \frac{2^n}{2n} \right\rfloor^+, \text{ for } 1 \leq \tau \leq L - 1 \]  

where \( [x]^+ \) denotes the smallest integer greater than or equal to \( x \). The left inequality follows from the second and third properties in (6); the right inequality is due to the peculiar features of De Bruijn sequences, that are full length sequences, a period of which includes all the possible binary \( n \)-tuples. In the case of binary De Bruijn sequences of span \( n = 5 \), the bound gives \( 0 \leq \max \theta(\tau) \leq 16 \).

The cross-correlation computed between pairs of De Bruijn sequences \( a \) and \( b \) randomly chosen, of the same span and period \( L \), denoted as \( r_{ab}(\tau) = \sum_{i=0}^{L-1} a_i b_{i+\tau} \), for \( 0 \leq \tau \leq L - 1 \), exhibits properties very similar to those discussed for the auto-correlation function:

\[ r_{ab}(\tau) = r_{ba}(L - \tau), \text{ for } 0 \leq \tau \leq L - 1 \]
\[ \sum_{\tau=0}^{L-1} r_{ab}(\tau) = 0 \]
\[ r_{ab}(\tau) \equiv 0 \mod 4, \text{ for } n \geq 2, \forall \tau \]

For the cross-correlation function of a pair of De Bruijn sequences \( a \) and \( b \) (\( a \neq b \)) of the same span \( n \), the following bound holds [16]:

\[ -2^n \leq r_{ab}(\tau) \leq 2^n - 4, \text{ for } 0 \leq \tau \leq L - 1 \]  

All the possible cross-correlation values are integer multiple of 4. Fig. 2 shows the average cross-correlation profile of binary De Bruijn sequences of span 5.

Fig. 2. Average cross-correlation profile of binary De Bruijn sequences of length 32