Introduction

The stone processing industry is experiencing a dramatic increase in the utilization of industrial diamond. Circular sawing with diamond-impregnated segments attached to a circular steel core is extensively used for cutting granite and other structural stones. With the growing use of natural granite as a construction material, there is increasing demand on reducing the sawing costs and increasing production efficiency.

Diamond impregnated segments on the saw periphery consist of randomly dispersed diamond abrasive grains embedded in a metal matrix. As sawing proceeds, the segments wear down and new diamonds emerge from the matrix. In general, the metal matrix and the diamond wear rates should be appropriately matched in order to facilitate efficient cutting and high wear resistance of the segments. The cutting behavior also depends on the processing parameters. A fundamental understanding of what happens during sawing is needed in order to provide a technological basis for enhancing the process efficiency.

The productivity achieved in sawing of granite is related to the mechanics of the process, as well as the wear resistance of the diamond segments. Forces, power, and energy have been extensively investigated for virtually all types of machining processes, including sawing and grinding. In general, the measured grinding power is equal to the product of the peripheral cutting velocity and the force component tangential to the wheel surface at the grinding zone [1]. For most shallow cut grinding, the tangential force component as in shallow cut grinding [3,4], whereas others have assumed fixed values for the position of resultant force [5,6]. However, in sawing experiments on granite conducted over a limited range of conditions [2], the location of the line of action of the resultant force was found to vary considerably along the contact length between the saw blade and the workpiece. Therefore the assumption of a fixed location for the resultant force is highly questionable and can lead to big errors in estimating the power.

A basic parameter in machining processes is the specific energy, which is defined as the energy expended per unit volume of material removal. The significance of the specific energy as a fundamental parameter derives from the fact that any proposed machining mechanisms must be able to account for the magnitude of the specific energy and its dependence on the operating parameters. As a practical matter, the magnitude of the specific energy is especially useful for estimating the power requirements for a particular cutting operation. Values of the specific energy for grinding and sawing of various granites have been reported from about 3.2 to 6.9 J/mm³ [2,7], although one study indicated much lower values for sawing of only about 0.5 J/mm² [8]. Unfortunately the results reported by different researchers are difficult to compare due to the relatively narrow range of sawing parameters in each case and differences in the granite workpiece and diamond tooling.

The present research was undertaken to investigate the forces, power, and specific energy for sawing of granite over a wide range of operating conditions. In addition to sawing, some grinding experiments were also conducted in order to extend investigation of stone cutting to much smaller removal rates.

Experimental

Circular sawing experiments were conducted in the down cutting mode as illustrated in Fig. 1 on an HPSM-I machine at National Huaqiao University. The saw blade of diameter d₁ at the interface between the abrasive tool surface and the workpiece [1,2]. Furthermore, the location of the resultant force vector may provide important insight regarding the force distribution along the interface and the prevailing grinding mechanisms [1]. In past studies on sawing forces, some researchers have erroneously taken the measured horizontal force to be the same as the tangential component as in shallow cut grinding [3,4], whereas others have assumed fixed values for the position of resultant force [5,6].
= 350 mm consisted of 24 equally spaced metal bonded abrasive segments (circumferential length 40 mm, width 3.2 mm, height 7 mm) containing 40/50 grit diamond with a concentration of 30 (7.5 percent by volume), which were brazed to the periphery of a circular steel core. The circumferential gap between adjacent segments at the blade periphery was approximately 5.8 mm, thereby giving a blade segment ratio (ratio of diamond segment length to total blade circumference) of \( \lambda = 0.85 \). Grinding experiments were conducted also in the down mode on a Brown & Sharpe 1236 Hi-Tech CNC grinder at the University of Massachusetts. A resin bonded diamond wheel (Norton Company, DN180 N100 B1/2) was used of diameter \( d_s = 305 \text{ mm} \) and width \( b = 12.7 \text{ mm} \) containing 180 grit diamond with a concentration of 100 (25 percent by volume). The workpiece material for both sawing and grinding was gray granite consisting of approximately 25 percent quartz of 1.0–1.5 mm grain size, 55 percent plagioclase, 15 percent alkaline feldspar, and 5 percent mica and other constituents. The material had a hardness \( H = 10 \text{ GPa} \), elastic modulus \( E = 50 \text{ GPa} \), and fracture toughness \( K_c = 1.9 \text{ MPa m}^{1/2} \).

All sawing and grinding tests were at a peripheral cutting velocity \( v_c = 28 \text{ m/s} \). The workpiece velocity \( v_w \) and wheel depth of cut \( a_p \) (see Fig. 1) were varied over a wide range of values giving specific removal rates (volumetric removal rates per unit width), \( Q_w' = v_w a_p \), ranging from 20 to 1050 mm/s for sawing and from 0.5 to 35 mm/s for grinding. For sawing, the cutting rate is more commonly expressed in terms of the area cut per unit time, which is identical to \( Q_w' \). City water was used as the cutting fluids for the sawing tests, and a 5 percent solution of heavy duty soluble oil was used for the grinding tests. The saw blade was trued and dressed before each test by “cutting” refractory bricks on the sawing machine. The diamond grinding wheel was dressed using a soft AlO₃ dressing stick before each grinding pass.

The active grain density \( C \) for the diamond segments and diamond wheels was obtained by counting active diamond grains observed on all the blade segments and on the wheel surface using an optical microscope. It was found that \( C = 0.7 \text{ mm}^2 \) for the saw blade and \( C = 21 \text{ mm}^2 \) for the grinding wheel.

The horizontal and vertical components, \( F_h \) and \( F_v \), acting on the saw blade and grinding wheel surfaces at the contact zone of length \( l_a \) (see Fig. 1) were measured using piezoelectric platform dynamometers (Kistler 9257BA for sawing and Kistler 9257A for grinding). The horizontal force component \( F_h \) was generally found to be in the positive direction for sawing with large depths of cut, and in the negative direction for shallow cut sawing and grinding. It should be noted that the definition of \((+)\) and \((-)\) adopted herein for \( F_h \) in Fig. 1 is typical of that for sawing, but opposite to that for grinding. For the sawing experiments, the force signals from the dynamometer were fed to a dynamic signal recorder (Di-2200) at a sampling frequency of 2.56 kHz and then filtered by Matlab on a PC with a cut-off frequency of 20 Hz. For the grinding experiments, the force signals from the dynamometer were collected on a PC after passing through an analog filter (Krohn-Hite 3322) to an A/D board at a sampling frequency of 300 Hz and being filtered by Matlab on a PC with a cut-off frequency of 20 Hz. The power was obtained for grinding using a Hall-effect transducer (F. W. Bell-PX 2202B) and for sawing by measuring the current. The net spindle power \( P_m \) was determined in each case by subtracting the idling power from the total power. From the net spindle power, the specific energy \( u \), which is defined as the energy per unit volume of material removal, was calculated as \( u = \frac{P_m}{Q_w'} \) where \( b \) is the engaged cutting width corresponding to the width of the saw blade segments or of the grinding wheel.

A separate series of sawing experiments was conducted to measure the radial blade wear. For this purpose, multiple sawing passes were taken at nominal specific removal rates of \( Q_w' = 250, 500, \) and 750 mm/s using various combinations of \( v_w \) and \( a_p \). The radial wear was measured on four equally spaced segments to within about 5 \( \mu \text{m} \). Variable speed was done at each test condition to obtain radial wear \( \Delta r_s \) of at least 50 \( \mu \text{m} \). Wear performance was then expressed in terms of the \( G \)-ratio (grinding ratio), which is the volumetric ratio of material removal to blade wear. Another related parameter particularly applicable to sawing is the wear performance factor \( W \), which is defined as the area of the workpiece product (volume of the total cutting length and depth of cut) to the radial wear of the saw blade. The relationship between \( W \) and the \( G \)-Ratio can be readily obtained as:

\[
W = (G \text{-RATIO}) \pi d_s \lambda \label{eq:2}
\]

where \( \lambda \) is the blade segment ratio (\( \lambda = 0.85 \)) as previously mentioned.

Granite surfaces were observed after sawing or grinding using a JEOL JSM-5410 scanning electron microscope (SEM). For these observations, the surfaces were coated with a carbon film.

Results

Experimental results are presented in Fig. 2 (sawing) and Fig. 3 (grinding) as plots of the vertical and horizontal force components per unit width, \( F_v' \) and \( F_h' \), power per unit width, \( P_v' \) and specific energy, \( u \), versus the specific removal rate \( Q_w' \). Due to the difficulty in accurately controlling the workpiece velocity in the sawing experiments, the indicated values of \( v_w \) in Fig. 2 are nominal values. The actual values, obtained from the duration of the recorded force signals, varied by as much as 10 percent from the indicated values. In all calculations involving \( v_w \), actual measured values were used.

For the sawing experiments in Fig. 2, the vertical force component increases monotonically with removal rate, linearly at smaller removal rates and with a progressively diminishing rate at larger removal rates. The results for all the workpiece velocities are close to the same curve. While the magnitude of the horizontal force component can also be seen to increase with removal rate, its magnitude depends on the workpiece velocity. With slower workpiece velocities, the rate of increase with removal rate is much steeper. It is interesting to note that the horizontal force component reverses direction and becomes negative at high workpiece velocities (shallow depths of cut). The measured power is approximately proportional to the removal rate, which would in-

Fig. 1 Illustration of sawing and grinding in down mode
dicate a nearly constant specific energy. However upon closer examination, it can be seen that the linear plot of power versus removal rate tends toward a small intercept value of about 17.5 W/mm. Consequently the specific energy in Fig. 2 tends to increase at small removal rates, and remains nearly constant at about 1.4 J/mm³ at larger removal rates. The trend of progressively increasing specific energy with decreasing removal rate is typical of many abrasive machining processes.

The situation for grinding in Fig. 3 is somewhat different. The vertical and horizontal grinding force components per unit width, $F_h$ and $F_v$, both increase linearly with removal rate. In this case, however, the direction of the horizontal component is negative (see Fig. 1), which corresponds to what is generally found with shallow cut grinding. Again in this case, the power is nearly proportional to removal rate. The corresponding specific energy increases somewhat at the smallest removal rates, and reduces to a constant “steady state” value of about 2.2 J/mm³ at larger removal rates.

**Force Analysis**

Now let us proceed to calculate the force components tangential and normal to the abrasive surface. The results indicate two possibilities for the direction of the horizontal force component for sawing, and one possibility for shallow-cut down grinding. This is schematically illustrated in Fig. 4 which shows the various force components exerted by the workpiece on the wheel at the contact zone for sawing and grinding. The location of the resultant force, which is indicated by the angle $k_f$, depends on the force intensity distribution along the contact zone. It is apparent that as the length of the contact zone approaches zero, which approximates the situation for shallow cut grinding, the tangential force component should become equal to the horizontal component, and the normal component to the vertical component. If this situation is not satisfied, it is not possible to accurately obtain the tangential and normal force components only from the measured horizontal and vertical components. In such a situation it is necessary to either measure the power, or to accurately know the resultant force location $k_f$ in Fig. 4. Since the power $P_m$ was measured, the tan-
The calculated tangential force per unit width, $F_t'$, for sawing and grinding is plotted versus the corresponding horizontal force per unit width, $F_h'$, in Fig. 5a for sawing and in Fig. 5b for grinding. It is clear that $F_t'$ is much different than $F_h'$ for the sawing experiments, but the two forces are almost identical for the grinding experiments as expected.

Results are shown in Fig. 6 for the tangential and normal sawing force components per unit width, $F_t'$ and $F_n'$, and their ratio, $F_t'/F_n'$. Analogous to the power in Fig. 2, $F_t'$ increases nearly linearly with $Q_w$. $F_n'$ also increases with $Q_w$, but at a progressively diminishing rate. The force ratio $F_t'/F_n'$ increases from about 0.06 at the lowest removal rate to about 0.15 at the highest. For grinding, the corresponding force ratio $F_t'/F_n'$ in Fig. 7 is comparable to the highest values obtained with sawing, although $Q_w'$ is much lower.

The results from the separate blade wear tests for sawing are summarized in Fig. 8 as plots of $G$-Ratio versus $Q_w'$ for various workpiece velocities. Values for the $G$-ratio range from about 6,000 to 12,000. Although there is considerable scatter in the results, the $G$-ratio appears to decrease slightly with $Q_w'$. The corresponding values of the wear performance factor $W$ (see Eq. (2)) range from about 7 m$^3$/mm to 14 m$^3$/mm.

The location of the resultant force along the contact zone in Fig. 4 is indicated by the angle $k \phi$, where $\phi$ is the total included angle of the contact zone given by [9]:

$$\phi = \cos^{-1}\left(1 - \frac{2a_p}{d_p}\right)$$  (4)

and $k$ represents the fractional angular distance from the bottom to the top of the contact zone. From geometrical considerations in Fig. 4, it can be readily shown that.
where ‘+’ is for the situation in Fig. 4a and ‘−’ for the situation in Fig. 4b,

\[ k = \frac{d + \beta}{\delta} \]  

(5)

and

\[ \delta = \tan^{-1}\left(\frac{F_t}{F_n}\right) \]  

(6)

\[ \beta = \tan^{-1}\left(\frac{F_n}{F_t}\right) \]  

(7)

In Fig. 9, results for \( k \) obtained from Eqs. (4)–(7) are plotted versus the contact length \( l_c \) between the sawblade and the workpiece, which can be approximated by its chord length \((a_d d_t)^{1/2}\) [9]. It can be seen that \( k \) increases with contact length from about 0.26 at the shortest length to about 0.65 at the longest. In other
words, the location of the resultant force is proportionally further away from the bottom of the cutting zone with longer contact lengths. In some cases, it has been assumed that the resultant force acts at the mid-point of the cutting zone \((k=0.5) \) [6,10], which can lead to significant errors in calculating \( F_t \) and \( F_n \) from \( F_h \) and \( F_v \). For creep feed grinding, it has been found that \( k=0.67 \) [1], which is consistent with a triangular force intensity distribution along the grinding zone apparently arising from the proportional variation of the instantaneous removal rate along the contact zone from a maximum at the top to zero at the bottom. For the grinding experiments, the depth of cut and length of the contact zone were so small that it was not possible to obtain reliable values for \( k \).

**SEM Observations**

SEM observations are presented in Fig. 10 for the sawn and ground workpiece surfaces. These pictures together with numerous other observations reveal extensive fracture accompanied by some localized ductile flow in the form of plowed striations along the cutting direction. The ground surfaces in Figs. 10c and 10d exhibit much more ductile flow than the sawn surfaces in Figs. 10a and 10b. Similar observations have been reported for grinding of ceramics [11].

**Discussion**

The forces, power, and specific energy are useful parameters for selecting efficient operating conditions for sawing, while taking into account the diamond abrasive consumption and machine capabilities. In the preceding analysis, it was seen that difficulties arise in analyzing the forces in sawing due to the orientation of the resultant force at the contact zone with the relatively large depths of cut. While the vertical and horizontal components are usually measured, it is the normal and tangential components which are of more fundamental significance.

The tangential force component is directly related to the power and specific energy, which will be analyzed at a later point in this paper. For understanding the implication of the normal force, it is useful to consider the grinding kinematics at the interface between the abrasive tool and the workpiece in Figs. 1 and 4. The rectilinear motion of the workpiece relative to the saw blade can be considered to cause a radial infeed velocity of the workpiece into the wheel periphery in a direction normal to the contact zone. This infeed velocity \( v_r \), which corresponds to the radial component of the workpiece velocity at the mid-point of the contact zone, can be written as [9]:

\[ v_r = \frac{v_w}{2} \]

![Fig. 10 SEM photographs of sawn and ground surfaces: (a) sawing, \( a_p = 30 \text{ mm} \) and \( v_w = 175 \text{ mm/s} \) (b) sawing, \( a_p = 30 \text{ mm} \) and \( v_w = 350 \text{ mm/s} \) (c) grinding, \( a_p = 0.015 \text{ mm} \) and \( v_w = 100 \text{ mm/s} \) (d) grinding, \( a_p = 0.010 \text{ mm} \) and \( v_w = 100 \text{ mm/s} \) ]
normal to the contact zone

Fig. 11 Average normal contact stress versus infeed velocity normal to the contact zone

\[ v_i = v_w \left( \frac{a_p}{d_s} \right)^{0.5} \]  \hspace{1cm} (8)

Figure 11 shows the average normal contact stress \( \sigma_n \) at the grinding zone plotted versus \( v_i \). For this purpose, \( \sigma_n \) obtained as:

\[ \sigma_n = \frac{F_n'}{C N l_c} \]  \hspace{1cm} (9)

where \( \lambda \) is the blade segment ratio and \( l_c \) is the length of the cutting zone approximated by its chord length \( (a_p d_s)^{1/2} \) [9]. The results show a monotonic increase in the normal contact stress with \( v_i \), which might be expected. However, this is contrary to other reported results, which appear to show the opposite effect over a limited range of test conditions [8]. It was suggested that such anomalous results could be attributed to the increased tendency for fracture at high removal rates.

For investigating the fracture tendency of both the workpiece and the diamond grains, a relevant parameter to consider is the average normal force, \( f_n \), on each diamond grain [12,13]. The average normal force per grain is obtained as:

\[ f_n = \frac{F_n'}{C N l_c} \]  \hspace{1cm} (10)

where \( C \) is the active grain density and \( l_c \) is the length of the cutting zone. This force can be related to the maximum grain depth of cut (also referred to as the maximum undeformed chip thickness) taken, on the average, by an abrasive grain as it passes through the cutting zone as illustrated in Fig. 12. The maximum grain depth of cut, which is a measure of the cutting severity at individual grains, can be written as [9]:

\[ h_m = \sqrt{\frac{3}{C} \lambda C \tan \theta} \sqrt{\frac{v_w}{v_s}} \sqrt{a_p} = \sqrt{\frac{3}{\lambda}} \sqrt{\frac{Q_w'}{v_s \lambda C \tan \theta \sqrt{d_s} \sqrt{a_p}}} \]  \hspace{1cm} (11)

where \( \theta \) is the semi-included angle of the active grain point which is assumed to be triangular. For continuous grinding wheels without slots, \( \lambda = 1 \) and is omitted. The addition of \( \lambda \) in Eq. (11) reflects the reduction in active grains due to the slots.

Figure 13 shows plots of \( f_n \) versus \( h_m \) for sawing and grinding. Values for \( h_m \) were calculated using \( \theta = 60^\circ \) degrees, with \( \lambda = 0.85 \) for sawing and \( \lambda = 1 \) for grinding. For sawing in Fig. 13a, \( f_n \) varies from about 2 N to 14 N, and \( h_m \) from about 10 \( \mu \)m to 45 \( \mu \)m. It is remarkable to see that all the results fall very close to a single straight line such that the normal force per grain is approximately proportional to the maximum grain depth of cut. This result can be directly applied in a practical way to estimating the normal force on the saw blade. The corresponding plot for grinding in Fig. 13b shows much finer cutting conditions with \( f_n \) less than 0.06 N and \( h_m \) less than 3 \( \mu \)m. These results for grinding might be combined with those for sawing, thereby extending the results for sawing to lower removal rates as indicated by the dotted line in Fig. 13a. However more data is needed at intermediate \( h_m \) values between 3 \( \mu \)m and 10 \( \mu \)m to ascertain how reasonable it might be to characterize the relationship between \( f_n \) and \( h_m \) for both sawing and grinding by a single curve.

It is also of interest to see how the normal force per grain affects wear of the saw blade segments. This can be inferred from the plot G-Ratio versus \( h_m \) as seen in Fig. 14. The available data at all three removal rates indicates that the G-Ratio is maximized at \( h_m \approx 33 \mu m \), which from Fig. 13a would correspond to a normal force per grain of \( f_n \approx 9 \) N. This behavior suggests a transition in the predominant diamond wear mechanism from attrition at a smaller normal force per grain, to fracture at a larger force per grain. In order to minimize diamond consumption, optimum operating conditions for sawing of this granite material with this particular type of diamond saw blade should be selected so that \( h_m \approx 33 \mu m \). Additional experiments are needed to validate the apparent wear transition and to ascertain whether this phenomenon might be generally applicable to sawing of granite.

Results for the specific energy for sawing and grinding are plotted versus \( h_m \) in Fig. 15. In both cases, the specific energy tends to increase at smaller grain depths of cut and is nearly constant at larger depths of cut, which is similar to what has been observed in grinding of ceramics. However the magnitudes of the specific energy observed here are somewhat smaller than what has been found for grinding of ceramics [11,12].

The magnitude of the specific energy and its dependence on the operating parameters and workpiece properties should be a direct consequence of the prevailing machining mechanisms. The SEM observations suggest material removal mainly by fracture, although there is also evidence of ductile flow. For grinding of other
brittle materials, including ceramics and glasses, it has been suggested that removal mechanisms should be dominated by high energy ductile flow at small values of $h_m$ when the normal load per grit is less than a critical load for lateral cracking to occur, and by brittle fracture at large values of $h_m$ above the critical load for lateral cracking. From a fracture mechanics analysis, an expression for the minimum normal threshold load $F_l^*$ for lateral cracking has been obtained as [14]:

$$F_l^* = \zeta \left( \frac{K_c}{H} \right) f \left( \frac{E}{H} \right)$$  \hspace{1cm} (12)

where $\zeta$ is a dimensionless constant and $f(E/H)$ is a weak function such that $\zeta f(E/H) \approx 2 \times 10^5$. Using values for $K_c$, $E$, and $H$ for the gray granite given above leads to $F_l^* \approx 2.6$ N. Values for $f_n$ in Fig. 13 are mostly greater than 2.6 N for sawing, but less than 2.6 N for grinding. This might suggest that brittle fracture should prevail for the sawing experiments and ductile flow for the grinding experiments. The SEM observations indicate brittle fracture for both sawing and grinding, although many more striations associated with ductile flow were observed for grinding than for sawing. It should be noted that the normal forces can vary significantly from grain to grain, so the average force is unlikely to be

![Fig. 13 Normal force per grain versus maximum undeformed chip thickness: (a) sawing, and (b) grinding](image1)

![Fig. 14 G-Ratio versus maximum undeformed chip thickness](image2)

![Fig. 15 Specific energy versus maximum undeformed chip thickness: (a) sawing, and (b) grinding](image3)
sufficient to characterize whether or not fracture occurs during each encounter with the workpiece. Furthermore, the grain depth of cut, as shown in Fig. 12, varies from zero to a maximum value during the engagement between an abrasive grain and the workpiece. For down cutting as in the present investigation, the maximum grain depth of cut occurs near the beginning of the engagement at the top of the contact zone, and thereafter decreases to zero as the abrasive grain disengages at the bottom of the cut.

If the interaction between the abrasive grains and the workpiece involved only brittle fracture, then the corresponding specific energy \( u_f \) could be estimated as the product of the surface area generated by fracture and the fracture energy per unit area of the material removal [15]. In order to make a rough estimate of \( u_f \), the particles removed by grinding can be assumed for simplicity to be small cubes of dimension \( b_f \). In this case, the total surface area produced per unit volume of material removed, \( a_f \), is equal to the total surface area of a cube divided by its volume:

\[
a_f = \frac{6b_f^2}{b_f} = 6 \tag{13}
\]

Approximating the fracture surface energy as half the energy release rate \( G_c (G_c = K_c^2/2E) \) for crack formation (two surfaces), the specific energy due to fracture becomes

\[
u_f = \frac{G_c}{2} a_f = \frac{3G_c}{b_f} \tag{14}
\]

From SEM observations, the smallest particles removed by diamond grains were found to be about 10 \( \mu \)m for sawing and 2 \( \mu \)m for grinding. Using these values for \( b_f \) leads to a specific fracture energy \( u_f = 1.8 \times 10^{-2} \text{ J/mm}^3 \) for sawing and \( u_f = 0.9 \times 10^{-2} \text{ J/mm}^3 \) for grinding. These values represent a negligible portion of the total specific energies in Fig. 13. Although brittle fracture is apparently responsible for most of the material removal for both sawing and grinding of granite, it can be concluded that most of the energy associated with these processes is expended in other ways.

Another mechanism which could account for some of the energy expenditure in sawing of granite is associated with the grinding swarf [6]. At high removal rates typical of sawing, the applied fluid together with the granite particles (swarf) removed by fracture form a slurry which is entrapped in the pores on the abrasive surface and interacts with the metallic bond matrix. An expression for the tangential force associated with the slurry, \( F_{sw} \), can be written as [6]:

\[
F_{sw} = k_{sw} R_{sw} \tag{15}
\]

where \( k_{sw} \) is a friction factor and \( P_{sw} \) is the pressure of the granite slurry against the bond material. Using a special sawing set-up with slurries but no actual cutting [6], values for \( k_{sw} \) were estimated as 0.25–0.45. Furthermore the slurry pressure \( P_{sw} \) over a limited range of conditions was found to be proportional to the volumetric ratio \( R_{sw} \) of material removal to available porosity on the surface of the abrasive segment:

\[
P_{sw} = k_{sw} R_{sw} \tag{16}
\]

An expression for the parameter \( R_{sw} \) can be readily obtained as [13]:

\[
R_{sw} = \frac{a_{sw} b}{v_{sw} \left( \pi d \right)^{-1} H_c} \frac{\pi l v_{sw} \cos \theta}{H_c} \tag{17}
\]

where \( H_c \) is the mean protrusion height of diamond grain. Combining Eqs. (15)–(17) to obtain \( F_{sw} \), the corresponding specific energy associated with the slurry can be written as:

\[
u_{sw} = \frac{F_{sw} v_{sw}}{Q_{sw} b} = \frac{\pi d \lambda k_{sw} k_{sw}}{H_c} \tag{18}
\]

Using the empirical value of 0.133 MPa obtained for \( k_{sw} \) [6] and approximating \( H_c \) as 25 percent of the diamond grain dimension leads to a value of \( u_{sw} = 0.37 \text{ J/mm}^3 \). This would suggest that about 30 percent of the sawing energy might be due to the slurry and its interaction with the matrix. Additional research is needed to show how the slurry, consisting of grinding particles and fluid, interacts with the matrix of bonded abrasive cutting tools.

The sawing swarf may also be responsible for the shift in the location of the line of action of the resultant force toward the exit at the bottom of the cut with shorter contact lengths as seen by the plot of \( k \) versus \( l_c \) in Fig. 9. When considering this effect, it is important to take into account the role of the slots in providing a place for swarf storage as the abrasive segments on the saw periphery abrade away the granite workpiece. In the absence of any slots, the swarf generated would have to be transferred through the entire contact length before being ejected at the bottom of the cut. In this case, the sawing accumulating toward the bottom of the cut could interfere with the cutting action of the diamond grains, thereby increasing the force intensity. It is interesting to note that the shortest contact lengths, where the resultant force location is shifted relatively the most toward the exit at the bottom of the cut, is slightly shorter than the saw blade segment length of 40 mm. In this case the slots may be relatively ineffective in removing swarf entrapped along the contact zone between the surface of the abrasive segment and the granite workpiece. With much shallower depths of cut, this should become much less of a problem because the contact lengths are much shorter thereby facilitating ejection of the entrapped swarf from the cutting zone. At the other extreme, the longest contact lengths in Fig. 9 are more than three times the saw blade segment length. Here saw ejection is apparently facilitated by the availability of more slots to store at least some of the swarf as the saw blade cutting surface passes through the cutting zone and subsequently ejects it. Since the localized instantaneous removal rate varies proportionally from zero at the bottom of the cut to a maximum at the top, a triangular force distribution would be expected if the force were simply proportional to the localized removal rate along the contact zone [9]. In this case, \( k \) would be equal to 2/3, which is about the same as what was found in the present study with the longest contact lengths.

While the SEM observations of the machined surfaces shows evidence of brittle fracture, virtually all of the energy associated with the sawing and grinding of granite must have been expended by other mechanisms. It appears that perhaps 30 percent of the energy for sawing may be associated with the grinding swarf as it interacts with the tool surface, which still leaves most of the energy unaccounted for. By analogy with recent results obtained for grinding of ceramics and glass [11], most of the energy expenditure may be attributed to ductile flow associated with the plowed grooves observed on the sawn and ground surfaces. Assuming that virtually all the grinding energy for ceramics is associated with ductile plowing, a plowing energy model was developed for grinding of ceramics which relates the grinding power per unit width \( P_m \) to the rate of plowed surface area (shown as \( A_g \) in Fig. 12 for a single undeformed chip) generated on the sides of the plowed grooves. Multiplying the plowed area for a single undeformed chip in Fig. 12 by the number of abrasive grains passing through the cutting zone, an expression for the plowed surface area generated per unit time per unit width was obtained as [11]:

\[
S' = \lambda C_v A_g \frac{\lambda C_v h_m l_c}{\cos \theta} \tag{19}
\]

In Fig. 16, results are presented with \( P'_m \) for grinding and \( P'_g \) for sawing plotted versus \( S' \) with \( \theta = 60 \) degrees. Whereas \( P'_m \) represents the total power per unit width, \( P'_g \) is the total power per unit width minus the apparent power per unit width associated with the swarf (\( P'_g = P'_m - u_{sw} a_{sw} P_{sw} \) with \( u_{sw} = 0.37 \text{ J/mm}^3 \)). Both curves show single valued nearly linear relationships between the power and the area generation rate. Similar results have been found for
Material removal for sawing and grinding of granite occurs mainly by brittle fracture, although most of the energy is expended by ductile flow due to plowing. About 30 percent of the sawing energy might be due to the swarf and its interaction with the bond matrix and the applied fluid. Analogous to recent results for grinding of ceramics and glass, linear relationships were obtained between the power per unit width and the rate of plowed surface area generated.

The tangential force component in abrasive sawing of granite is much different than the measured horizontal force component due to the large depth of cut, but the two forces are almost identical for grinding. The location of the resultant force component is relatively further away from the bottom of the cutting zone with longer contact lengths, which might be attributed to interference of sawing swarf with the cutting process.

The undeformed chip thickness is a useful parameter for characterizing the cutting process for granite. The normal force per grain was approximately proportional to the undeformed chip thickness. Optimum sawing conditions should be selected at an intermediate undeformed chip thickness where the G-Ratio was found to reach a maximum value.

Future work is planned with different types of granite and saw blades in order to explore the general validity of the present findings. For selection of optimal working conditions, it will be of particular interest to see how the granite type and saw blade composition affect the G-Ratio and the normal force per grain. Other efforts will focus on achieving a better understanding of how the swarf interacts with the abrasive segments and the applied fluid.

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References


Fig. 16 Power per unit width versus rate of plowed surface area generated per unit width

\[ J_s = 8.0 \times 10^3 \text{ J/m}^2 \]  
\[ (\text{grinding}) \]

\[ J_s = 10.4 \times 10^3 \text{ J/m}^2 \]  
\[ (\text{sawing}) \]